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AVAILABLE FROM UTC Coordinator, Technical Education Research Center--Southwest, 4201 Lake Shore Dr., Waco, TX 76710 (Concept Modules, \$0.80 each; Application and Supplementary Modules, \$0.40 each; Notebook for Modules, \$2.25; Instructor's Guide, \$8.00)

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ABSTRACT

This concept module on potential and kinetic energy is one of thirteen modules that provide a flexible, laboratory-based physics instructional package designed to meet the specialized needs of students in two-year, postsecondary technical schools. Each of the thirteen concept modules discusses a single physics concept and how it is applied to each energy system. This module describes potential and kinetic energy and discusses the conservation of energy in mechanical, fluidal, electrical and thermal systems. Included with the concept module are twenty application modules that discuss a specific application of a single concept in a single energy system, and contain a student laboratory exercise. Laboratory modules included are Flywheel, Conservation of Energy in Collisions, Kinetic Energy and Horsepower of an Automobile, Energy Stored in Large Tanks, Venturi Meters, Pitot Tubes, Energy Conversion in Fluid Systems, Gas Laws, Siphons, Energy Stored in a Battery, Energy Stored in a Capacitor, Energy Stored in an Inductor, Series and Parallel Capacitors, Heat Storage Systems, Electrical Equivalent of Heat, Mechanical Equivalent of Heat, Thermal Drying, and Mixing Liquids at Different Temperatures. (LRA)

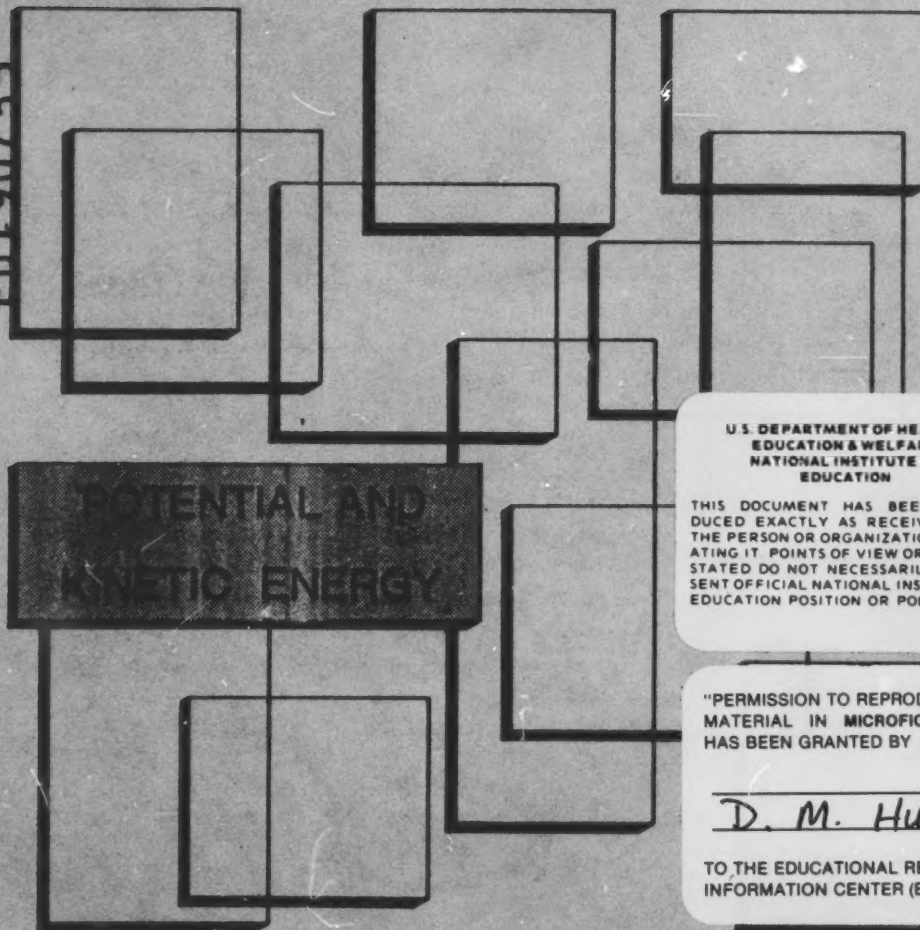
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UTC

UNITED TECHNICAL CONCEPTS

CONCEPT MODULE 7-0

FD190755



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INTRODUCTION TO UTC

Complex, modern equipment often contains combinations of hydraulic, pneumatic, thermodynamic or optical subsystems, as well as mechanical and electrical components. These subsystems and components appear not in isolation but as interdependent parts of complex systems. Consequently, a modern technician must understand not only the separate subsystems but also their interrelationships and the roles they play in an integrated system.

Since physics provides the foundation for modern technology, physics instruction in technical programs cannot be overemphasized. Physics courses for technicians should achieve three major objectives:

- Provide students with all the basic physics principles that apply to their chosen field
- Reinforce these principles through the use of "real" examples and hands-on experience.
- Organize knowledge and skills gained in a manner that provides ease of application in solving systems-oriented problems.

These objectives must be accomplished in a minimum of instructional time, and in a manner flexible enough to meet the needs of a variety of technical specialties.

Unified Technical Concepts (UTC) is a flexible, laboratory-based physics instructional package designed to meet the specialized needs of students in two-year, post-secondary technical schools. UTC organizes technical physics into 13 basic concepts (Force, Work, etc.) that apply in four "energy systems" (Mechanical, Fluidal, Electrical, Thermal). Each of the 13 "concept" modules discusses a single physics concept and how it is applied to each energy system. Similarly, each "application" module discusses a specific application of a single concept in a single energy system, and contains a student laboratory exercise.

The recommended prerequisite for UTC is one year of high school algebra, but eight supplementary modules are available for students requiring additional skills or review in the areas of math, units and graphing.

UTC is an alternate method of presenting what heretofore has been a formidable and sometimes confusing subject for technicians. It provides laboratory experiments that apply to specific technological fields, which allows a student to relate more quickly to module content - to understand how it applies to a specific technology.

UTC also represents a unique approach to teaching applied physics. Its flexible modular format allows it to be tailored by physics and technology instructors to meet

specific needs of students in a specific technology.

STRUCTURE OF UTC

The modern world is based on the control and application of energy. Technology is the means by which that energy is applied to accomplish a desired task. Technical physics, being primarily a study of the physical basis of the control and use of energy, can be categorized according to the type of energy involved. Thus, in UTC, physics is divided first into four "energy systems" - Mechanical, Fluidal, Electrical and Thermal.

The same divisions are characteristic of traditional approaches to teaching technical physics. A student usually begins by studying mechanics. Once this branch of physics (energy system) is mastered, electromagnetism, thermodynamics and hydraulics are studied (one energy system at a time).

UTC takes a different approach.

CONCEPTS IN PHYSICS

The basic ideas, or "concepts," of physics appear in all energy systems, and these concepts relate to one another in similar ways in all systems. UTC unifies technical physics by presenting first the physics concept and then its application in each energy system.

The first concept studied, for example, is that of force, - the physical quantity that produces motion. The basic characteristics of force are followed by a discussion of how this concept applies to "force-like quantities" in the following energy "systems":

- In mechanical systems force and torque cause motion of mass.
- In fluidal systems pressure difference causes motion of fluids.
- In electrical systems potential difference causes motion of mass.
- In thermal systems temperature difference causes motion of heat energy.

Problems, demonstrations and hands-on laboratory exercises are used to show the application of the concept in each energy system, and to specific devices or processes common in modern technology.

Thus, UTC provides a student with the basic concepts of physics reinforced by practical application and organized into a consistent whole; UTC is readily applied to future technical problems, whether in other courses or on the job.

COMPONENTS OF UTC

UTC incorporates three types of modules.

CONCEPT MODULES

Thirteen concept modules form the backbone of UTC (each presents a single concept in applied physics and discusses the application of that concept in each energy system):

- 1-0 Force.
- 2-0 Work.
- 3-0 Rate.
- 4-0 Momentum.
- 5-0 Resistance.
- 6-0 Power.
- 7-0 Potential and Kinetic Energy.
- 8-0 Force Transformers.
- 9-0 Energy Convertors.
- 10-0 Transducers.
- 11-0 Vibrations and Waves.
- 12-0 Time Constants.
- 13-0 Radiation.

APPLICATION MODULES

Each of the 170 application modules discusses the application of a single physics concept in a single energy system, and each module includes a laboratory exercise in which students investigate the application. For example, 5E5 "Resistance of Wires" (located under Concept 5-0 "Resistance"), discusses the electrical resistance of wires. In the laboratory, students calculate and measure experimentally the resistance of various wires.

SUPPLEMENTARY MODULES

In UTC, mathematics has been reduced to a minimum: one year of high school algebra is adequate preparation. The following eight supplementary modules are available for students requiring additional preparation or review:

- S1 Formula Interpretation.
- S2 International System of Units.
- S3 Logarithms and Exponents.
- S4 Angles and Triangles.
- S5 Vectors and Scalars.
- S6 Dimensional Analysis.
- S7 Reading and Drawing Graphs.
- S8 Precision, Accuracy, and Measurement.

STRUCTURE OF A UTC MODULE

Each module in UTC contains seven basic elements:

INTRODUCTION

A brief description of the topics to be discussed, and their importance in technology, introduces each module.

PREREQUISITES

A list of prerequisite UTC modules, and notice of any special knowledge or skills required for successful completion of the module, are given.

OBJECTIVES

Specific tasks that a student should be able to accomplish after completing the module are stated in behavioral terms; objectives also are the basis of testing for UTC.

DISCUSSION

This segment is an explanation of the topic to be studied. It contains the appropriate physics as well as examples of its application. The discussion includes development of equations, example problems, figures and tables of useful data. It addresses each of the objectives (except those pertaining directly to a laboratory experiment).

LABORATORY (called DEMONSTRATIONS in the Concept Modules)

The laboratory section of UTC modules consists of the following three parts:

- "Materials." A list of all equipment and supplies required for the laboratory.
- "Procedure." A step-by-step set of instructions for performing the experiment.
- "Data." An organized format for recording all necessary data and observations.

SELF CHECK

A set of questions and problems based on the module objectives is included. It is intended for use by students to check their own understanding of module content.

REFERENCES

This list of sources directs students to additional information on topics discussed in the module.

7

TABLE OF CONTENTS

7-0	Potential and Kinetic Energy
7M1	Piledriver
7M2	Potential Energy of a Spring
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INTRODUCTION

Concept Module 2-0, "Work," described work as the product of a displacement like quantity and a force like quantity. Energy was defined as "the ability to do work." Energy can be contained in a system in a variety of forms. Most of the world's energy needs are supplied today by the burning of fossil fuel to convert their stored chemical energy to thermal energy. Solar energy is electromagnetic radiation emitted by the sun. This module discusses two important forms of energy in technology and demonstrates further the unification of physics. Kinetic energy is the energy of moving mass; it is understood most easily in mechanical and fluidal systems, although heat energy actually is the kinetic energy of moving molecules. Potential energy is the energy that exists due to position or potential difference. Potential energy involves no motion, but it has the capacity to produce motion at some later time. The Principle of the Conservation of Energy states, "Energy can be neither created or destroyed." The total energy in any closed system remains constant, although it may change in form. This module describes potential and kinetic energy and discusses the conservation of energy in mechanical, fluidal, electrical and thermal systems.

PREFREQUISITES

Student should have completed Concept Module 6-0, "Power."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define potential energy, kinetic energy, and conservation of energy by using examples from mechanical systems.
- Given any two of the quantities in the following groups, determine the third:

Mass, velocity, kinetic energy.

Mass, height, potential energy.

Spring constant, spring displacement, potential energy.

Moment of Inertia, angular velocity, kinetic energy.

Capacitance, voltage, potential energy.

- Given Bernoulli's Equation and the height of liquid in a tank, determine the exit velocity at the bottom of the tank if there is no fluid friction.
- List and describe the three processes that transfer thermal energy.
- Discuss the conservation of energy as it applies to fluidal, electrical, and thermal systems.

DISCUSSION

Module 2-0, "Work," presented the concept that work done on an object represents the amount of energy transferred to that object. Work was defined in each energy system as "the product of a force-like quantity and a displacement-like quantity." Work is done when a force-like quantity acts to cause motion in a system. Energy is the ability to do work. The presence of energy in a system does not mean that force-like quantities are producing changes, but only that there is a potential for that to occur in the future.

Work is done unless energy is transferred. The energy may change forms, as when friction changes mechanical energy into thermal energy; or it may retain the same form, as when a moving object strikes a stationary one, causing it to move. In all cases, the total energy of the system is conserved.

MECHANICAL SYSTEMS

TRANSLATIONAL

In many cases, the work done on an object appears primarily as energy of motion (kinetic energy). Total kinetic energy of a moving object is related to its mass and velocity by Equation 1. The sample problem (Example A) illustrates the use of Equation 1.

$$E_K = 1/2 mv^2 \quad (1)$$

where

E_K = kinetic energy.

m = mass.

v = velocity.

EXAMPLE A: KINETIC ENERGY OF A MOVING BALL

Given: A 250-gram ball moves with a velocity of 15 m/s.

Find: Amount of kinetic energy it possesses.

Solution: From Equation 1 —

$$E_K = 1/2 (0.25 \text{ kg})(15 \text{ m/s})^2$$

$$E_K = 28.1 \text{ joules.}$$

$$E_K \geq (\text{kg})(\text{m/s})^2$$

$$\geq \left(\frac{\text{kg} \times \text{m}}{\text{s}^2} \right) (\text{m})$$

(Recall: $\frac{\text{kg} \cdot \text{m}}{\text{s}^2} = \text{newton (N).}$)

NOTE: When only units are substituted into the equation, kinetic energy is shown to have the same units as work (i.e., Joules).

$$\geq N \times m$$

$$E_k \geq \text{joules}$$

When work is one to lift an object, the work becomes potential energy of the object (Figure 1).

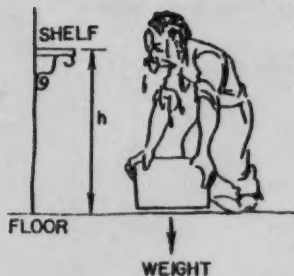


FIG. 1

To lift the box to the shelf in Figure 1, the man must exert a force equal to the weight of the box. We know that the work done by the man is equal to Wh (where W = work, and h = height from floor to shelf). This value is the exact amount of increase in potential energy of the box after it is placed on the shelf. When the box is placed on the shelf, the gravitational potential energy of the box with respect to the floor is expressed by Equation 2.

$$E_p = Wh \quad (2)$$

where

E_p = potential energy.

W = work.

h = height of shelf from floor.

Since weight of an object is related to its mass by the gravitational constant according to $W = mg$, Equation 2 often is given in the form of Equation 3.

$$E_p = mgh \quad (3)$$

where

g = gravitational constant.

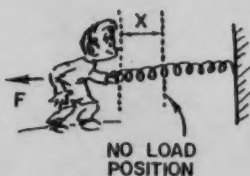


FIG. 2

Gravitational potential energy is an important and useful concept. There are means of storing energy other than the raising of weight; for example, a stretched or compressed spring waiting to be released possesses potential energy.

Work necessary to stretch the spring beyond its "no load" position (Figure 2) is given by $W = Fx$. In this case, F is not a constant force, but increases as x increases.

Potential energy of the spring stretched x units beyond its normal length is given in Equation 4.

$$E_p = \frac{1}{2} kx^2 \quad (4)$$

where

k = spring constant (depends upon the mechanical stiffness of the spring; see below).

x = units to which spring is stretched beyond its normal limits.

The work put into the spring (Work = Fx) is stored by the stretched spring as potential energy ($E_p = \frac{1}{2} kx^2$). The value of k is just the magnitude of the force required for unit stretch of the spring; for example, 5 lb/inch. For a given spring, the value of k does not vary, provided the spring is not stretched beyond its elastic limit (i.e., the point at which the spring will not return to the no-load position if force is removed).

Concept Module 5-0, "Resistance," shows that resistance opposes motion and eventually causes moving objects to slow down or stop. Resistance cannot be completely eliminated from a system, even with the best lubrication. However, friction often is omitted, as a matter of convenience, when energy systems are studied. Friction will be neglected, therefore, throughout the remainder of this module. Obviously, the conclusions reached are only good approximations of "real world" situations in which friction plays a large part.

When no energy is lost to friction, the sum total of the kinetic and potential energy of a system remains constant. Thus, if kinetic energy of the system decreases, the potential energy of that system increases an equal amount. This concept is amplified in Example B.

EXAMPLE B: POTENTIAL ENERGY OF STORED WATER

Given: Two tons of water are at the top of a 200-foot waterfall, ready to go over the edge.

Find: (a) The amount of potential energy that the water possesses with respect to the bottom of the falls,
(b) the amount of potential and kinetic energy the water has when it has fallen halfway down and
(c) the amount of kinetic energy the water has when it has reached the bottom.

Solution: (a) At top of falls -

$$\begin{aligned} E_p &= mgh = Wh \\ &= (4000 \text{ lbs})(200 \text{ ft}) \\ E_p &= 800,000 \text{ ft}\cdot\text{lbs} \\ E_k &= 0 \end{aligned}$$

(b) At halfway point -

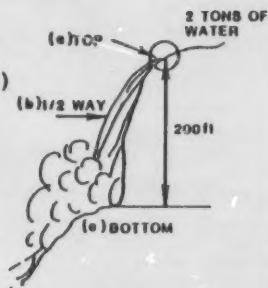
$$E_p = (4000 \text{ lbs})(100 \text{ ft}) \\ = 400,000 \text{ ft}\cdot\text{lbs}$$

$$E_k + E_p = 800,000 \text{ ft}\cdot\text{lbs} \\ \therefore E_k = 400,000 \text{ ft}\cdot\text{lbs}$$

(c) At bottom -

$$E_p = 0 \\ E_k = 800,000 \text{ ft}\cdot\text{lbs}$$

We have assumed that the sum of $E_k + E_p$ is the same throughout.



CONSERVATION OF ENERGY

The fact that the sum of potential and kinetic energy remains unchanged, even while there is a transformation of one form of energy to the other, is a demonstration of the Principle of the Conservation of Energy. This principle is one of the cornerstones upon which all of science and technology is built.

Consider the changes in kinetic and potential energy of a roller coaster car as it travels on the track illustrated in Figure 3.

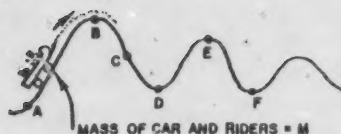


FIG. 3

From point A up to B, energy must be supplied to the car by an electric winch. Assume the winch is released at B. At point B, energy supplied by the electric winch has been stored in the car as potential energy. This energy is equal to mgh , as given by Equation 3. As the car coasts down the first hill past point C toward D, its energy is changing rapidly

from potential to kinetic. As kinetic energy increases, so does the car's velocity (speed), according to Equation 1 ($E_k = 1/2 mv^2$). The car always is moving faster at low points on the track (points D and F) and slower at high points (B and E). However, at any point the sum total of its kinetic and potential energy is equal to the original amount of energy supplied to the car during ascent from A to B. Friction, if considered, gradually steals some of the energy away.

A spring with an attached mass bouncing from its end is another example of a system with changing kinetic and potential energy (Figure 4).

Work is done on the system in Figure 4 when the spring (with mass attached) is stretched from position A to B, a distance equal to x . The amount of energy necessary to do this work is given by Equation 4 ($E_p = 1/2 kx^2$). When the spring is released, it moves upward through the rest position. At the rest position, the displacement x is zero; and potential energy $1/2 kx^2$ equals zero at this point. However, as the mass passes the rest point, it is moving with its greatest speed and, thus, has its greatest kinetic energy. As the mass rises above its rest position, its speed is reduced, but its gravitational potential energy and spring potential energy are increased. This process continues until the highest point is reached, where its velocity and kinetic energy are zero, but its gravitational potential energy and spring potential energy are maximum. At any point during the cycle, total energy of the mass is given as follows:

(potential energy stored in spring) +
(kinetic energy) + (gravitational
potential energy), or —

$$1/2 kx^2 + 1/2 mv^2 + mgh$$

This process would continue forever if friction did not dissipate the energy gradually and reduce the vertical oscillation.

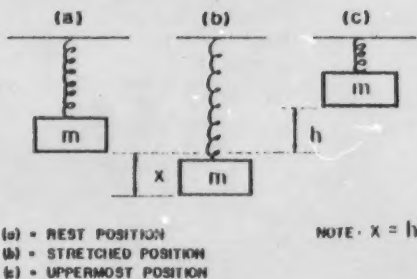


FIG. 4

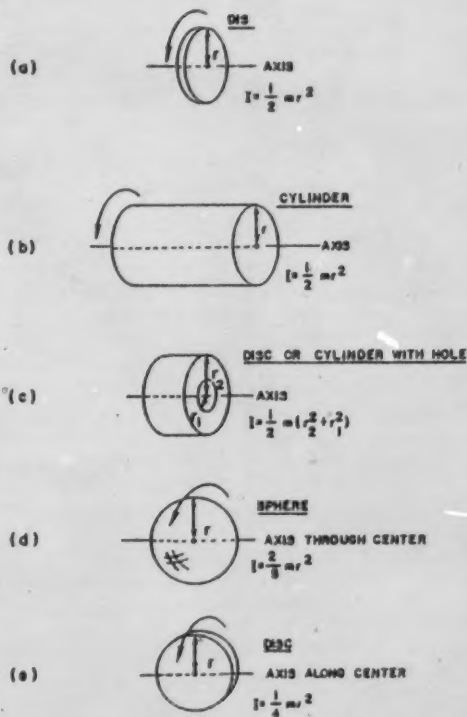


FIG. 5

ROTATIONAL

Concept Module 4-0, "Momentum," introduced the concept of moment of inertia and suggested that it is the quantity in a rotational system that corresponds to mass in a linear system. Recall that the moment of inertia (I) varies with the shape of the object and with the position of its axis of rotation. Formulas for determining the moment of inertia of various objects are summarized in Figure 5.

Since the moment of inertia corresponds to mass in a linear system, and since angular velocity (ω , rad/s) corresponds to the angular rate of motion, the reasonable assumption is that the formula for rotational kinetic energy is similar in form to the formula for linear kinetic energy. Thus, $1/2 mv^2$ becomes $1/2 I\omega^2$ when going from linear to rotational motion. Rotational kinetic energy is given in Equation 5.

$$E_K = 1/2 I\omega^2 \quad (5)$$

where

I = moment of inertia.

ω = angular velocity in rad/s.

Two problems that illustrate the use of Equation 5 are given in Examples C and D.

EXAMPLE C: KINETIC ENERGY OF A FLYWHEEL

Given: A 20-kg flywheel disc whose radius is 25 cm rotates about its axis with an angular velocity of 300 r/min (rpm).

Find: Its kinetic energy.

Solution: $E_K = 1/2 I\omega^2$
 $I = 1/2 mr^2$ (from Figure 5a)
 $= (1/2)(20 \text{ kg})(0.25 \text{ m})^2$
 $I = 0.625 \text{ kg} \times \text{m}^2$
 $E_K = 1/2 (0.625 \text{ kg} \times \text{m}^2)(300 \frac{\text{r}}{\text{min}} \times \frac{\text{min}}{60\text{s}} \times \frac{2\pi \text{ rad}}{\text{r}})^2$
 $= 1/2 (0.625)(987) \frac{\text{kg} \times \text{m}^2}{\text{s}^2}$
 $E_K = 308 \text{ joules}$

EXAMPLE D: ENERGY USED TO START A LAWN MOWER

Given: A manual starting system on a lawn mower requires a force of 180 newtons to pull the cord. The homeowner pulls the rope a total of 0.8 meters when attempting to start the mower. The flywheel around which the rope is wound has a radius of 12 cm and a mass of 7 kilograms.

Find: (a) This moment of inertial of the flywheel,
(b) work put into pulling the cord, and
(c) kinetic energy and angular velocity given the flywheel, assuming no load conditions.

Solution: (a) $I = \frac{1}{2} mr^2$
 $= \frac{1}{2} (7 \text{ kg}) (0.12 \text{ m})^2$
 $I = 0.05 \text{ kg} \times \text{m}^2$



(b) Recall from Module 2-0 — or alternately —
 $W = \tau \theta$ $W = Fx$
 $\tau = FL$ $= (180 \text{ N}) (0.8 \text{ m})$
 $= (180 \text{ N}) (0.12 \text{ m})$ $= 144 \text{ Nm}$
 $\tau = 21.6 \text{ Nm}$ $W = 144 \text{ joules}$

$\theta = \frac{s}{r}$
 $= \frac{0.8 \text{ m}}{0.12 \text{ m}}$
 $\theta = 6.67 \text{ rad}$
 $W = (21.6 \text{ Nm}) (6.67 \text{ rad})$
 $W = 144 \text{ joules}$

(c) $E_K = \frac{1}{2} \omega^2$

If no friction or load is present, all the work put into pulling the rope is transferred into kinetic energy of the flywheel. Thus, the kinetic energy = 144 joules.

$144 \text{ joules} = \frac{1}{2} (0.05 \text{ kg} \times \text{m}^2) \omega^2$
 $\omega^2 = \frac{2 (144)}{0.05} \frac{\text{rad}^2}{\text{s}^2}$
 $\omega^2 = 5760 \frac{\text{rad}^2}{\text{s}^2}$
 $\omega = 76 \frac{\text{rad}}{\text{s}}$

An object can be rotating at the same time that it is moving forward. If this is the case, the object has both rotational kinetic energy and translational kinetic energy. Total kinetic

energy of the object would be found by determining each energy separately and then adding the results, as illustrated in Example E.

EXAMPLE E: KINETIC ENERGY OF A BOWLING BALL

Given: A 7.2-kg bowling ball with a radius of 12 cm is thrown with a velocity of 4 m/s.

Find: Total kinetic energy of the ball before it strikes the pins.



Solution: Linear kinetic energy

$$E_K = \frac{1}{2} mv^2$$

$$= \frac{1}{2} (7.2 \text{ kg}) (4 \text{ m/s})^2$$

$$E_K = 57.6 \text{ joules}$$

Rotational kinetic energy

$$E_K = \frac{1}{2} I \omega^2$$

$$I = \frac{2}{5} mr^2 \text{ for a sphere}$$

$$= \frac{2}{5} (7.2 \text{ kg}) (0.12 \text{ m})^2$$

$$= 0.04 \text{ kgm}^2$$

$$= \frac{1}{2} (0.04 \text{ kgm}^2) (33.3 \text{ rad/s})^2$$

$$E_K = 22.2 \text{ joules}$$

Recall:

$$\omega = \frac{v}{r}$$

$$= \frac{4 \text{ m/s}}{0.12 \text{ m}}$$

$$\omega = 33.3 \frac{\text{rad}}{\text{s}}$$

$$E_{K\text{total}} = E_{K\text{linear}} + E_{K\text{rotational}}$$

$$= 57.6 \text{ joules} + 22.2 \text{ joules}$$

$$E_{K\text{total}} = 79.8 \text{ joules}$$

FLUIDAL SYSTEMS

Daniel Bernoulli (1700-1782) applied the conservation-of-energy principle to fluid motion. The outcome of his work is known as "Bernoulli's principle," which has wide-ranging practical applications. Devices, such as an automobile carburetor, a venturi meter, a sailboat in motion, a curving baseball, and the lift on an airplane wing, all utilize Bernoulli's principle.

Since liquids are practically incompressible, use of volume rather than mass to measure a moving liquid usually is more

convenient. Therefore, kinetic and potential energy formulas first must be modified to include volume rather than mass. This modification is accomplished, first, by recalling the definition of density ρ .

$$\rho = \frac{m}{V}$$

or,

$$m = \rho V$$

Thus, mass can be replaced in the energy equations with density times volume. The results are given in Equations 6 and 7.

$$E_K = 1/2 (\rho V) v^2 \quad (6)$$

$$E_P = (\rho V) gh \quad (7)$$

where

g = acceleration of gravity (SI: 9.8 m/s^2 ; English: 32 ft/s^2).

Consider a fluid flowing in the pipe shown in Figure 6. Assume that the pipe has changing cross sections and does not all lie on the same level, and that the fluid flow is without resistance (zero viscosity) so that no fluid friction exists.

In a given amount of time, equal volumes of liquid will flow through section a and section b of the pipe if the fluid is incompressible.

Consider the work done in moving this volume V from section a to section b. Net pressure on the fluid in section a is $p_a - p_b$. Since the fluid in section a is moved through the volume V , the work done on the fluid is given by Equation 8.

$$W = (p_a - p_b) V \quad (8)$$

where

p_a = pressure at section a.

p_b = pressure at section b.

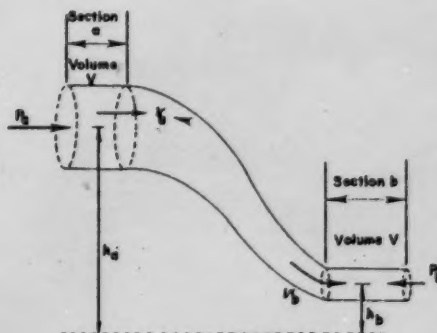


FIG. 8

Change in the E_K of the volume V of fluid is given by Equation 9.

$$\Delta E_K = 1/2 (\rho V) v_b^2 - 1/2 (\rho V) v_a^2 \quad (9)$$

where

v_a = fluid velocity at section a.
 v_b = fluid velocity at section b.

Change in the E_p of the volume V of fluid is given by Equation 10.

$$\Delta E_p = (\rho V) gh_b - (\rho V) gh_a \quad (10)$$

where

h_a = height of section a.
 h_b = height of section b.

If energy is to be conserved, the work done can show up only as a change in kinetic and potential energies of the volume of fluid. Thus -

$$W = \Delta E_K + \Delta E_p$$

$$(p_a - p_b) V = 1/2 (\rho V) v_b^2 - 1/2 (\rho V) v_a^2 + (\rho V) gh_b - (\rho V) gh_a$$

This arrangement is divided through by V and the terms rearranged to yield Equation 11.

$$p_a + 1/2 \rho v_a^2 + \rho gh_a = p_b + 1/2 \rho v_b^2 + \rho gh_b \quad (11)$$

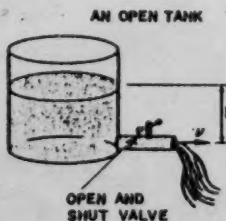


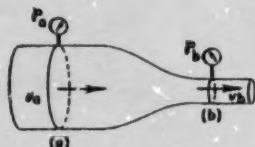
FIG. 7

Equation 11 is Bernoulli's equation and, as pointed out earlier, is a direct result of the conservation of energy. It states that the quantity $(p + 1/2 \rho v^2 + \rho gh)$ must be the same at any cross section of the flow pipe. If the flow is horizontal, then Bernoulli's principle leads to the "rule of thumb" that "when the velocity of a moving fluid is high, its pressure difference is low, and when the velocity is low, the pressure difference is high."

Example F given an application of Bernoulli's equation.

EXAMPLE F: MEASURING PRESSURE CHANGES IN A PIPE WITH VARYING DIAMETERS

Given: Water flows through the horizontal pipe shown, with a velocity at "a" of 4 m/s and at "b" of 10 m/s. The pressure gage at point "a" reads $250 \times 10^3 \text{ N/m}^2$.



Find: The pressure-gage reading at point "b".

Solution: Since the center of the pipe stays at the same elevation, potential energy terms in Equation 11 are equal, and Equation 11 reduces to the following:

$$p_a + 1/2 \rho v_a^2 = p_b + 1/2 \rho v_b^2$$

Substitution of the proper values into the above equation yields -

(Recall that ρ for water equals 1 g/cm^3 or 1000 kg/m^3 .)

$$250 \times 10^3 \frac{\text{N}}{\text{m}^2} + 1/2 (1000 \frac{\text{kg}}{\text{m}^3}) (4 \frac{\text{m}}{\text{s}})^2 = p_b + 1/2 (1000 \frac{\text{kg}}{\text{m}^3}) (10 \frac{\text{m}}{\text{s}})^2$$

$$[250 \times 10^3 + 8 \times 10^3] \frac{\text{N}}{\text{m}^2} = p_b + 50 \times 10^3 \frac{\text{N}}{\text{m}^2}$$

$$[258 \times 10^3 - 50 \times 10^3] \frac{\text{N}}{\text{m}^2} = p_b$$

$$p_b = 208 \times 10^3 \frac{\text{N}}{\text{m}^2} \text{ (newtons/square meter)}$$

Consider a tank filled with fluid (Figure 7).

When the valve at the bottom of the tank is opened, fluid escapes with velocity v . Bernoulli's equation can be applied to the cross section at the upper surface of the reservoir and at the point of outflow.

At the upper surface, pressure is atmospheric pressure p_a , and velocity is negligible, $v_a = 0$. The left side of Equation 11 is then -

$$p_a = \rho gh$$

At the exit pipe, pressure again is atmospheric pressure, and height $h = 0$. Then the right side of Equation 11 is as follows:

$$p_a = 1/2 \rho v^2$$

Bernoulli's principle requires the left- and right-hand quantities be equal:

$$p_a + \rho gh = p_a + 1/2 \rho v^2 \quad (12)$$

$$\rho gh = 1/2 \rho v^2$$

Solving for v -

$$v^2 = 2 gh$$

or

$$v = \sqrt{2 gh} \quad (13)$$

The conservation-of-energy principle has allowed determination of the exit velocity of a fluid from a tank system. This is illustrated in Example G.

EXAMPLE G: CALCULATING EFFLUX VELOCITY

- Given:** A tank filled with 15 feet of water is equipped with a valve at the bottom, as shown in Figure 7.
- Find:** (a) Velocity of efflux. (b) If the spigot is aimed upward, how high will the water rise? (c) If the water actually rises less than the amount predicted, how can this be accounted for?
- Solution:** (a) $v = \sqrt{2 gh}$
 $= \sqrt{2 (32 \text{ ft/s}^2) (15 \text{ ft})}$
 $= \sqrt{960}$
 $v = 31 \text{ ft/s}$
- (b) If the spigot were pointed upward, kinetic energy of the water would be turned back into potential energy, and the water should rise to the original 15-foot height.
- (c) If the water does not actually rise 15 feet, the discrepancy can be explained in the following way:

Friction within the fluid and of fluid moving against walls of the pipe and valve has consumed a portion of the energy.

ELECTRICAL SYSTEMS

Consider now the concept of potential energy in electrical-energy systems as illustrated by the electrical circuit in Figure 8. The circuit consists of a capacitor connected to a battery.

A capacitor consists of two conducting plates separated by an insulating material. The battery in Figure 8 provides a potential difference V that forces electrons toward the capacitor. The charge that reaches the plate of the capacitor cannot move through the insulating material to reach the other plate. Opposite and equal charge builds up on the capacitor plates according to the plus (+) and minus (-) polarity shown in Figure 8. A stage will be reached when the battery cannot force any more charge onto the capacitor plates.

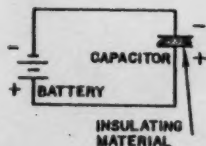


FIG. 8

The quantity of charge that builds up on either plate when the potential difference is one volt is called "capacitance." Capacitance depends upon area of the capacitor plates, distance between the plates, and the type of insulating material between them. Capacitance is said to be one farad when a one-volt battery deposits a charge of one coulomb on each capacitor plate. The definition of capacitance is given mathematically in Equation 15.

$$C = \frac{Q}{V} \quad (15)$$

where

- C = capacitance.
- Q = charge stored.
- V = voltage.

The farad is a very large unit of capacitance; therefore, capacitors frequently are rated in microfarads ($\mu F = 10^{-6} F$) or micro-microfarads ($\mu\mu F = 10^{-12} F$). A micro-microfarad is also referred to as a "picofarad" ($pF = 10^{-12} F$).

When a capacitor is charged in the manner described earlier, the charge remains on the capacitor for a time, even after the battery is removed. This stored charge represents a kind of potential energy. Anyone who accidentally touched a charged capacitor can verify the fact that energy is stored in the device. Energy stored by a capacitor is very similar, in principle, to the energy stored in a stretched spring.

Equation 16 provides a way to determine the amount of energy stored in a capacitor.

$$E_p = 1/2 QV \quad (16)$$

From Equation 15, $Q = CV$; and if this is substituted into Equation 16, two more additional useful forms for potential energy result:

$$E_p = 1/2 \frac{Q^2}{C} \quad (17)$$

$$E_p = 1/2 CV^2$$

Example H illustrates the use of Equations 15 and 17.

EXAMPLE H: FUNCTION OF A CAPACITOR

Given: Twelve volts are applied to charge a 0.5- μ F capacitor.

Find: (a) Amount of charge stored on the plates.
(b) Amount of potential energy stored by the capacitor.
(c) What becomes of the energy if the plates are shorted together.

Solution: (a) $C = \frac{Q}{V}$

or

$$\begin{aligned} Q &= CV \\ &= (0.5 \times 10^{-6} \text{ F})(12 \text{ V}) \\ Q &= 6 \times 10^{-6} \text{ coulombs} \end{aligned}$$

$$\begin{aligned} \text{(b) } E_p &= 1/2 CV^2 \\ &= 1/2 (0.5 \times 10^{-6} \text{ F})(12 \text{ V})^2 \\ E_p &= 3.6 \times 10^{-5} \text{ joules} \end{aligned}$$

(c) When the capacitor is shorted, a current flows in the connecting wire until the charges are neutralized. This current produces heat energy. A spark probably will occur. Some energy is given off as light when a spark occurs. Thus, most of the energy is dissipated as heat energy, and some is given off as light energy.

HEAT SYSTEMS

Up to this point, the discussion of kinetic energy has been easy to visualize in a moving object. Heat energy is not as easily "seen" as other forms of energy.

However, kinetic energy is involved in heat energy, because heat is nothing more than the sum total of all the kinetic energy possessed by the atoms of the object. The temperature of a substance is a measure of the average kinetic energy of its atoms.

When heat is transferred to or from an object, there is a corresponding change of temperature. The amount of transferred heat energy can be determined with the help of Equation 18.

$$H = mc\Delta T \quad (18)$$

where

H = thermal energy added to a body to cause a temperature change ΔT .

ΔT = temperature change of the body.

m = mass of the body.

c = constant characteristic of the body, known as the "specific heat" of the substance.

Recall that, by definition, water requires 1 calorie of heat energy to raise the temperature of 1 gram by 1 degree Celsius. Specific heat of water is then 1 cal/g $\times$ $^{\circ}\text{C}$. A specific heat such as 0.22 for aluminum means that only 0.22 a calorie is needed to change the temperature of one gram of aluminum one degree Celsius. Specific heats for other materials easily are found in standard reference handbooks.

Heat energy (molecular kinetic energy) can be transferred by three different ways: conduction, convection and radiation.

When two objects at different temperatures are put in contact, the rapidly-moving molecules of the hot object collide with, and transfer energy to, the molecules of the cold object. This is the basic process of heat conduction. When liquids and gases are heated, the hot material expands, causing a reduced density. The lighter, hot material floats upward, causing convection currents. In this way, heat energy is transported by convection. All objects that have a temperature above absolute zero emit heat energy in the form of electromagnetic radiation, and radiation is the only process by which heat energy can be transmitted through a vacuum. This phenomenon will be discussed in greater detail in Concept Module 13-0, "Radiation."

SUMMARY

In this discussion of energy, there is a striking similarity between certain mathematical representations of energy, which reveals analogies between physical systems. For example, the similarity of kinetic energy $\frac{1}{2}mv^2$ in the translational and $\frac{1}{2}I\omega^2$ in the rotational mechanical systems reveals the analogous roles in the two system played by v and ω (ratelike quantities) and by m and I (mass or inertiallike quantities.) Also, the similarity of potential energies $\frac{1}{2}kx^2$ and $\frac{1}{2}(1/C)Q^2$ in the mechanical and electrical systems reveals the analogous roles played by x and Q (displacementlike quantities) and by k and $1/C$.

Energy itself is a fundamental unifying concept, because it is common to all systems. In all systems we notice that whenever work is done, there is an increase in kinetic or in potential energy, or in both. When energy is transformed from one system to another, we expect the total amount of energy present to remain the same, including heat energy produced by friction. If friction is present, a portion of the work is represented by a frictional heat energy loss. In the absence of friction, the sum total of the kinetic and potential energy remains constant, although there may be exchanges between these two forms of energy. This fact, known as the "conservation of energy," is one of the fundamental principles upon which all science and technology is based.

DEMONSTRATIONS

MECHANICAL

MATERIALS

Inclined track.
Metal sphere, cylinder and ring or other similar objects.
Meter stick.
Stopwatch.

PROCEDURE

The principle of the conservation of energy will be applied to several objects rolling down an incline. Potential energy of the objects will be measured at the top of the incline, and translational and rotational kinetic energy will be calculated at the bottom of the incline.

1. Arrange a ball, cylinder, or ring at top of an inclined track. Measure height of object; calculate its potential energy from equation $E_p = mgh$.

$$mass = \underline{\hspace{2cm}}$$

$$g = \underline{\hspace{2cm}}$$

$$h = \underline{\hspace{2cm}}$$

$$E_p = \underline{\hspace{2cm}}$$

2. As object rolls, it will lose potential energy and gain kinetic energy (both rotational and translational) according to the principle of conservation of energy:

$$mgh = 1/2 I\omega^2 + 1/2 mV^2.$$

3. Measure v at bottom of incline. To accomplish this measurement, allow object to roll on a friction-free level surface after it comes off incline. Record time t required for object to travel a premarked, precisely-measured, level distance d ; calculate v from the following equation:

$$v = \frac{d}{t}$$

4. Calculate and record translational kinetic energy of object.

$$K = \underline{\hspace{2cm}}$$

$$v = \underline{\hspace{2cm}}$$

$$E_K = 1/2 mv^2 = \underline{\hspace{2cm}}$$

5. Calculate angular velocity ω from linear velocity v , using this equation:

$$\omega = \frac{v}{r}$$

Here, r is radius of the rolling object. Calculate rotational kinetic energy, $1/2 I\omega^2$, using a value for I from Figure 5 for shape of the object used. (A ball would be easiest.)

$$r = \underline{\hspace{2cm}}$$

$$\omega = \frac{v}{r} = \underline{\hspace{2cm}}$$

$$I = \underline{\hspace{2cm}}$$

$$E_K = 1/2 I\omega^2 = \underline{\hspace{2cm}}$$

6. Compare total kinetic energies as found in Steps 4 and 5 with original potential energy as found in Step 1. Is energy conserved, as stated in Step 2? Calculate a % difference.

$$\% \text{ difference} = \frac{E_P - (E_K)_{\text{total}}}{E_P} \times 100$$

7. Repeat entire procedure with other shaped objects as time permits.

FLUIDS

MATERIALS

Pail with hole in side near bottom.
Vernier caliper.
2-liter beaker or other volume measure.

PROCEDURE

The exit velocity of a fluid will be obtained in two different ways and the two values compared.

1. Obtain a pail with a small, drilled, circular hole in side near bottom. Fill pail to a line near top. Allow water to discharge from pail without restriction. Adjust water entering pail in such a manner that water level stays at same elevation throughout experiment (Figure 9).

2. Measure height h of water in pail, and calculate exit velocity from $v = \sqrt{2gh}$, as described in text.

$h =$ _____

$v =$ _____

3. To verify that this equation is valid, also obtain v in the following way: Recall from earlier modules that volume flow rate of a fluid $\Delta V/\Delta t$ is equal to velocity v of the fluid times the area A through which the fluid is moving.

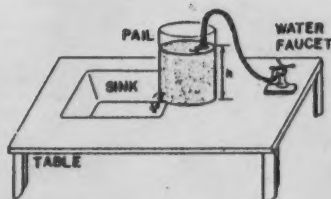


FIG. 9

$$\frac{\Delta V}{\Delta t} = Av$$

4. Record time necessary to fill a 2-liter beaker or other known volume with water coming from the hole. Calculate flow rate in cm^3/s . _____
5. Using a vernier caliper or other precision measuring tool, determine drilled diameter of hole and calculate its area in cm^2 .
6. With equation given in step 3 above, calculate velocity of discharged fluid.

$$\Delta V/\Delta t = \text{_____} \text{ cm}^3/\text{s}$$

$$A = \text{_____} \text{ cm}^2$$

$$v = \text{_____} \text{ cm/s}$$

7. Compare the two values obtained for the exit velocity by finding percentage variation?

$$\% \text{ variation} = \frac{\text{difference}}{\text{average}}$$

ELECTRICAL

MATERIALS

Small hand-operated d.c. generator.
Capacitor 10 μF , 200 volt.
Voltmeter.
Ammeter.
Spring scale.

PROCEDURE

The following values will be determined: (1) mechanical energy necessary to rotate a small electric generator used to charge a capacitor, and (2) efficiency of the system.

1. A hand-operated d.c. generator is necessary for this experiment. If only an a.c. generator is available, place a diode or rectifier in circuit to change a.c. to d.c.

2. Connect circuit as shown in Figure 10.

3. Calculate mechanical energy input from the work equation $W = Fx$. It will be necessary to use a spring scale on end of crank to determine force necessary to turn generator.

$$F = \underline{\hspace{2cm}} \text{ newtons}$$

Turn generator uniformly (counting revolutions) until a full charge is obtained on capacitor. When a full charge is obtained, no current will flow through ammeter. From number of revolutions N and radius of crank r , determine distance x moved to obtain a full charge.

$$x = 2\pi rN \text{ (meters)}$$

$$\text{Work Input} = Fx$$

$$\text{Work Input} = \underline{\hspace{2cm}} \text{ Joules}$$

4. Calculate potential energy stored in capacitor from equation $E_p = 1/2 CV^2$, as described in text.

$$E_p = \underline{\hspace{2cm}} \text{ Joules}$$

5. Considering energy stored in capacitor as the work output, calculate efficiency of this system:

$$\text{efficiency} = \frac{\text{work output}}{\text{work input}}$$

$$\text{efficiency} = \underline{\hspace{2cm}}$$



FIG. 10

SELF CHECK

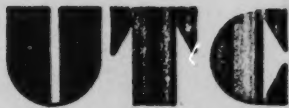
1. Define potential energy, kinetic energy, and conservation of energy, using examples from mechanical systems.

2. An automobile weighing 2800 lb is traveling at 55 miles per hour. What is its kinetic energy?
3. What would the speed of the automobile in problem 2 be if its kinetic energy were doubled?
4. An 1800-lb elevator is lifted a vertical distance of 70 ft by an electric motor. Calculate potential energy of the elevator at the 70-ft height.
5. A spring requires a force of 25 N to stretch it 15 cm. What is the spring constant (k)? How much energy is stored in the spring when it is stretched 45 cm?
6. An electrical capacitor of 1.5 μF is connected to a 6.0-V battery. How much potential energy is stored in the capacitor?
7. A solid wheel (disk) with a mass of 5 kg has a speed of 20 m/s. Determine its total kinetic energy.
8. Starting with Bernoulli's equation, derive the expression for the velocity of a frictionless fluid flowing from an opening in a large tank; and use this equation to determine the velocity of water exiting through a hole 50 ft below the water level of a tank.
9. Explain similarities and differences between conduction, convection, and radiation.
10. Discuss conservation of energy as it applies to mechanical, fluidal, electrical, and thermal systems.

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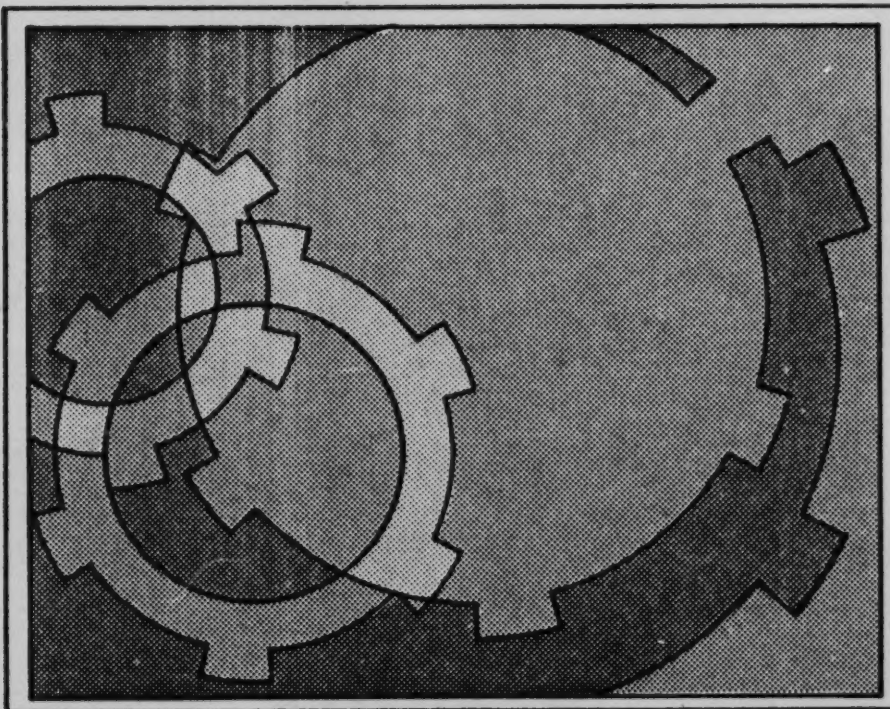
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UNIFIED TECHNICAL CONCEPTS

MODULE 7 M 1

PILEDRIIVER



MECHANICAL APPLICATION OF
POTENTIAL AND KINETIC ENERGY

Technical Education Research Center-Southwest

TEC - SW

INTRODUCTION

Many useful devices depend in part upon the conversion of gravitational potential energy to kinetic energy. In systems such as water wheels, this kinetic energy is converted to other forms before being used. In other devices, the energy may be delivered directly in one swift blow. The piledriver and the forging hammer are examples of this latter kind of mechanical system.

In this module, the piledriver as an energy storage and delivery device will be investigated. A small-scale piledriver constructed in the laboratory will be used in an investigation to determine the effects of changing the mass and height of a piledriver blow.

PREREQUISITES

Student should have completed Concept Module 7-0, "Potential and Kinetic Energy," and Application Module 3M3, "Displacement at a Constant Acceleration."

OBJECTIVES

To complete this module, the student should be able to:

- Calculate potential energy of a known mass at a known height.
- Calculate velocity of a mass upon impact after it has been dropped from a given height.
- Calculate kinetic energy of a known mass traveling at a known velocity.
- Construct a model piledriver in the laboratory and describe in a sentence or two the effects of changing the mass used and the height from which it is dropped.

DISCUSSION

Figure 1 shows one form of a piledriver. The mass is raised by a hoist to height h . It is then released and falls to strike the pile and drive it into the ground.

This is actually a device for storing a quantity of mechanical energy for later application as a brief pulse of mechanical energy. The engine of the hoist is incapable of producing energy at a rate sufficient to drive the pile continuously. The energy it does produce is stored as gravitational potential energy by using the hoist to raise a mass. This potential energy is converted to kinetic energy when the mass is dropped and accelerates under the influence of gravity. When the mass strikes the pile, its kinetic energy is applied in one brief, high-power pulse. The resulting downward force is of short duration but very high magnitude, and the pile is driven into the ground.

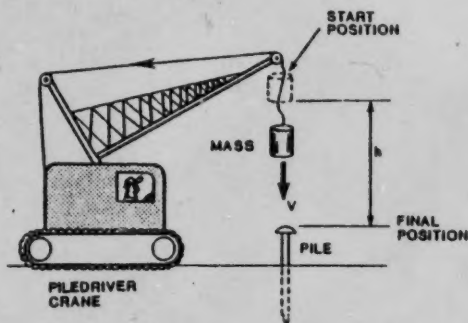


Fig. 1

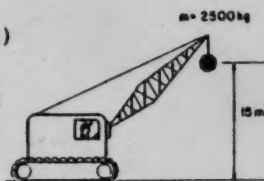
The amount of energy stored by the raised mass is given by Equation 1.

Gravitational potential energy = mass $\times$ acceleration of gravity $\times$ height.

In symbols: $PE = mGh$.

(1)

Examples A and B show how this equation is used to solve typical problems.



EXAMPLE A: CALCULATION OF GRAVITATIONAL POTENTIAL ENERGY

Given: A mass of 2500 kg raised to a height of 15 m.

Find: Its gravitational potential energy.

Solution: $PE = mGh$
 $= (2500 \text{ kg})(9.8 \text{ m/s}^2)(15 \text{ m})$
 $= 3.675 \times 10^5 \text{ kg-m}^2/\text{s}^2$
 $\therefore PE = 3.675 \times 10^5 \text{ Joules.}$

EXAMPLE B: POWER CALCULATION OF A HOIST

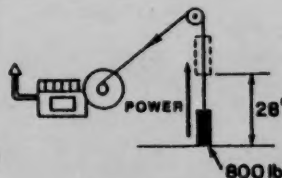
Given: A hoist raises a 800-lb weight to a height of 28 feet in 32 seconds.

Find: Power of the hoist.

Solution: $PE = mgh$
 $= (800 \text{ lb}) \times (28 \text{ ft})$
 $PE = 22,400 \text{ ft-lb.}$

$$\begin{aligned} \text{Power} &= \frac{PE}{\text{time}} \\ &= \frac{(22,400 \text{ ft-lb})}{(32 \text{ s})} \\ &= 700 \frac{\text{ft-lb}}{\text{s}} \times \left(\frac{1 \text{ horsepower}}{550 \text{ ft-lb/s}} \right) \end{aligned}$$

$\therefore \text{Power} = 1.27 \text{ horsepower.}$



When the mass is allowed to fall, all of the potential energy is changed to kinetic energy. Losses due to such factors as rope friction and air resistance are insignificant. Kinetic energy of the mass at impact is given by Equation 2.

$$\text{Kinetic energy} = \frac{1}{2} \times \text{mass} \times \text{square of velocity.}$$

$$\text{In symbols: } KE = \frac{1}{2} mV^2. \quad (2)$$

From Application Module 3M3, "Displacement at a Constant Acceleration," the velocity of a falling body after a fall of height h is given by Equation 3.

$$\text{Velocity} = \text{square root of } (2 \times \text{acceleration of gravity} \times \text{height}).$$

$$\text{In symbols: } V = \sqrt{2 gh}. \quad (3)$$

Examples C and D show the use of Equations 2 and 3 in solving problems.

EXAMPLE C: CALCULATION OF IMPACT VELOCITY AND KINETIC ENERGY

Given: The 2500-kg mass in Example A is allowed to fall 15 m to the ground.

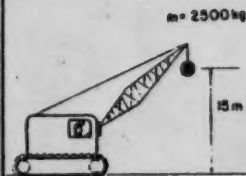
Find: Its velocity and kinetic energy on impact.

Solution:

$$\begin{aligned}\text{Impact velocity: } V &= \sqrt{2 Gh} \\ &= \sqrt{2 (9.8 \text{ m/s}^2) (15 \text{ m})} \\ &= \sqrt{294 \text{ m}^2/\text{s}^2} \\ \therefore V &= 17.15 \text{ m/s.}\end{aligned}$$

$$\begin{aligned}\text{Kinetic velocity: } KE &= \frac{1}{2} mV^2 \\ &= \frac{1}{2} (2500 \text{ kg}) (17.15 \text{ m/s})^2 \\ \therefore KE &= 3.675 \times 10^5 \text{ J.}\end{aligned}$$

Note: Same as the potential energy in Example A.



EXAMPLE D: CALCULATION OF IMPACT VELOCITY AND KINETIC ENERGY

Given: The 800-lb weight of Example B falls 28 ft.

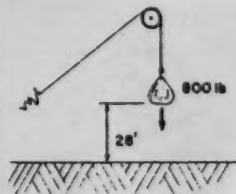
Find: Its velocity and kinetic energy.

Solution:

$$\begin{aligned}\text{Velocity: } V &= \sqrt{2 Gh} \\ &= \sqrt{2 (32 \text{ ft/s}^2) (28 \text{ ft})} \\ &= \sqrt{1792 \text{ ft}^2/\text{s}^2} \\ V &= 42.34 \text{ ft/s.}\end{aligned}$$

$$\begin{aligned}\text{Kinetic energy: } KE &= \frac{1}{2} mV^2 \\ &= \frac{1}{2} (25 \text{ slugs}) (42.34 \text{ ft/s})^2 \\ &= 22,400 \text{ slugs ft}^2/\text{s}^2 \\ KE &= 22,400 \text{ ft-lb.}\end{aligned}$$

Note: Same as PE in Example B.



Forging hammers and piledrivers are pure applications of the conversion of gravitational potential energy to kinetic energy. Striking tools such as sledge hammers and axes employ the same basic principle, but in these cases, part of the original potential energy lies within the muscles that propel the tool downward with an acceleration greater than that of gravity alone.

In all such devices, potential energy is converted to kinetic energy to deliver high-power pulses of mechanical energy. One question that remains to be answered is, what happens to that energy after the blow is struck? All of this energy is present in the dropped mass or the object struck as increased kinetic energy of the molecules, resulting in a temperature rise as an added benefit in such applications as metal forging.

LABORATORY

MATERIALS

Soft wooden block (pine) 2 inch x 4 inch x 4 feet.

One-meter section of clear plastic pipe, 2-1/2 inch inner diameter.

Two-inch diameter steel bar stock cut to length for three test weights of 0.5 kg, 1.5 kg and 3.0 kg.

Meter stick.

String sufficiently strong to support a 5.0-kg weight.

One-half inch galvanized roofing nails (large flat head).

Hammer.

Metric balance.

PROCEDURE

1. Use a hammer to start nail into wooden block. It should be driven in no farther than is necessary to ensure it will not fall over. Measure and record height h_1 of nail head above wood block.
2. Place plastic pipe on wooden block with nail at its center as shown in Figure 2. Tie string around smallest steel mass; position it inside pipe at selected height, h_2 above head of nail.
3. Release string, allowing mass to fall and strike nail. Remove mass and pipe; measure and record new height h_3 of head of nail.
4. Repeat above process until each mass has been dropped from five different heights. Use a new nail and a new spot on wood for each trial weight.
5. Complete data table.
6. Draw a graph showing penetration versus stored potential energy (Figure 3). Draw a separate line on graph for each drop weight; label each line.
7. Should graph be a straight line? Do you think resistance offered by wood to penetrating nail is same per unit depth of wood penetrated? If it were, would graph then be a straight line?

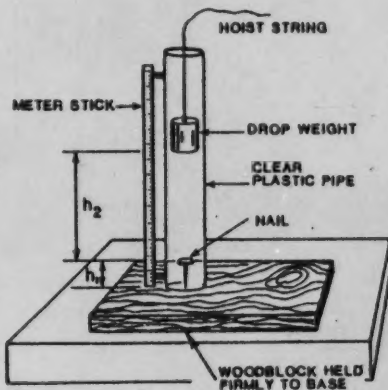


Fig. 2

DATA TABLE						
Data					Results	
Trial	Mass (kg)	Drop Height of Mass h_2 (m)	Original Height of Nail Head h_1 (cm)	Final Height of Nail Head h_3 (cm)	Stored Potential Energy (J) $PE = mgh_2$	Penetration $p = (h_1 - h_3)$ (cm)
1	(0.5)					
2						
3						
4						
5						
6	(1.5)					
7						
8						
9						
10						
11	(3.0)					
12						
13						
14						
15						

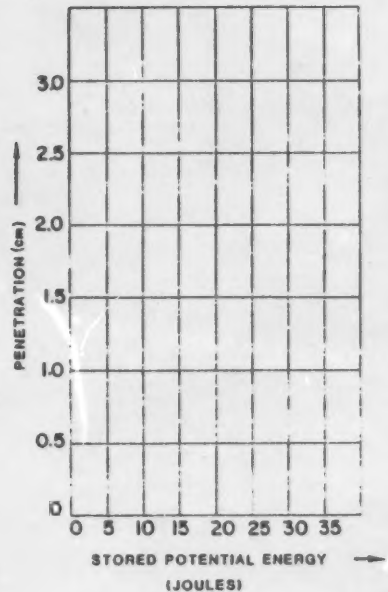


Fig. 3

SELF CHECK

1. What is the potential energy of each of the following masses or weights at the indicated heights?
 $m = 100 \text{ kg}$, $h = 2 \text{ m}$.
 $wt = 50,000 \text{ lb}$, $h = 4 \text{ ft}$.
 $m = 2850 \text{ kg}$, $h = 16.8 \text{ m}$.
 $wt = 2100 \text{ lb}$, $h = 35 \text{ ft}$.
2. Calculate velocity upon impact for each of the above masses or weights dropped from the given height.
3. Calculate kinetic energy of each of the masses upon impact.

4. Which of the masses used in the experiment makes most efficient use of the stored potential energy?

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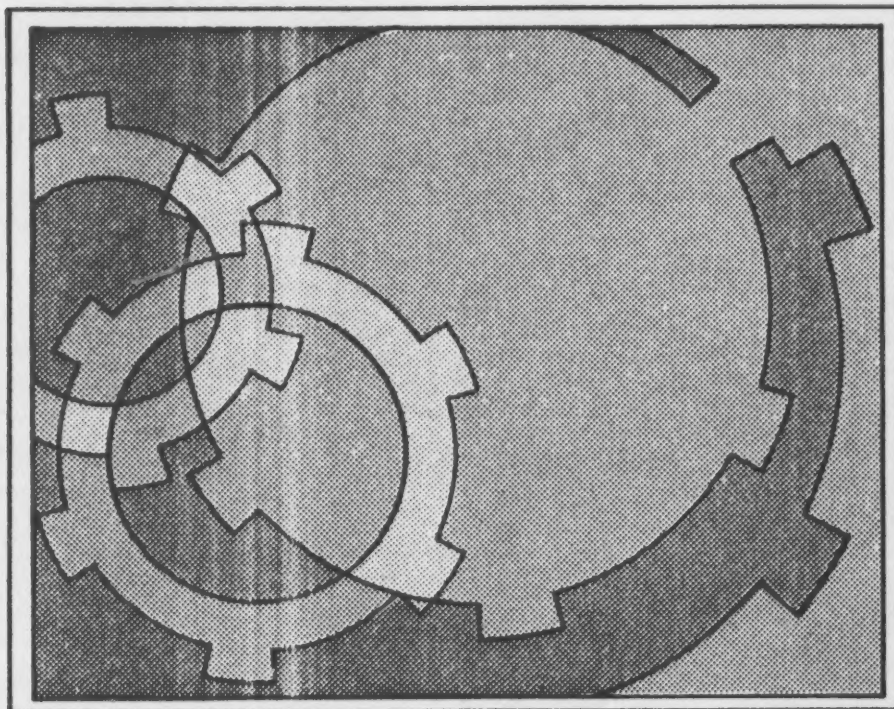
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UNIFIED TECHNICAL CONCEPTS

MODULE 7 M 2

KINETIC ENERGY OF A SPRING



MECHANICAL APPLICATION OF POTENTIAL AND KINETIC ENERGY

Technical Education Research Center-Southwest

TERC - SW

INTRODUCTION

The stretching or compressing of a spring stores mechanical energy in the spring. This energy can be recovered as useful work by allowing the force present in the spring to cause motion toward the equilibrium position of the spring. This module discusses the forces acting upon a spring and the storage of energy in it. In the laboratory, the student will determine a spring constant by direct measurement and by measurement of the period of a spring-mass system.

PREREQUISITES

Student should have completed Concept Module 7-0, "Potential and Kinetic Energy."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the following terms:

Restoring force.
Spring constant.
Hooke's law.

- Show graphically the relationships between spring force, displacement, and energy stored in a spring.
- Given any two of the following quantities, calculate the third:

Spring constant.
Displacement.
Spring force.

- Given any two of the following quantities, calculate the third:

Spring constant.
Displacement.
Stored energy.

- Show graphically the relationships between displacement and potential and kinetic energy in a spring-mass oscillator.
- Given any two of the following quantities for a spring-mass oscillator, calculate the third:

Spring constant.

Mass.

Period.

- List three applications of springs, and discuss how the spring acts as an energy-storage device in each case.
- Given the appropriate equipment, determine the spring constant of a spring by measurement of force and displacement and by measurement of the period of a spring-mass oscillator.

DISCUSSION

HOOKE'S LAW

The force provided by a stretched or compressed spring is described by Hooke's law (Equation 1).

$$F = -kx \quad (1)$$

where

- F = spring force.
- k = spring constant.
- x = displacement from equilibrium position.

Figure 1a illustrates a spring in its equilibrium position. Both displacement from equilibrium and from force are zero. In Figure 1b, the spring has been stretched to positive displacement x_1 . The force of the spring is F_1 , a negative quantity, since it is in the opposite direction from x_1 . In Figure 1c, the spring is compressed to a negative displacement x_2 , and the force of the spring is a positive F_2 . The force of the spring is called a "restoring force" because its action always is exerted to restore the system to the equilibrium position ($x = 0$). The minus sign in Equation 1 indicates that the force is always in the opposite direction from the displacement.

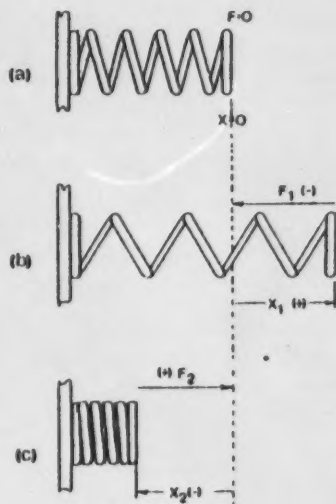


FIG. 1

Equation 1 may be rearranged to obtain Equation 2.

$$k = -\frac{F}{x} \quad (2)$$

This equation reveals the significance of the spring constant k , which is the ratio of the force of a spring to its displacement. The units, therefore, are newtons per meter (N/m).

The spring constant also is the slope of the force versus displacement graph, as shown in Figure 2. A larger value of k indicates that more force is required to stretch the spring.

Example A illustrates the use of Equation 2 in solving a problem.

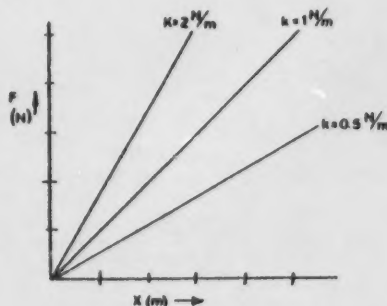


FIG. 2

EXAMPLE A: DETERMINING A SPRING CONSTANT

Given: A 10-lb weight is suspended from a spring and stretches the spring 6 inches.

Find: The spring constant.

Solution: Choose "up" to be positive.

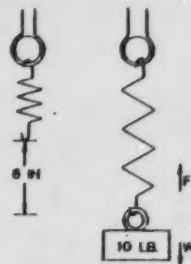
Spring force: $F = 10 \text{ lb}$
(upward)

Displacement: $x = -0.5 \text{ ft}$
(downward)

$$k = -\frac{F}{x}$$

$$= -\frac{10 \text{ lb}}{(-0.5 \text{ ft})}$$

$$k = 20 \text{ lb/ft}$$



WORK DONE ON A SPRING

In Concept Module 2-0, "Work," work was defined as force times displacement. Figure 3 is a graph of forces applied to compress a spring, as a function of displacement.

Vertical distance on the graph represents force, and horizontal distance represents displacement. Thus, work done on the spring is equivalent to area on the graph. The work done to compress the spring to a displacement of x_1 is indicated by the shaded area on the graph. This area is the area of a triangle with a base x_1 and a height $F_1 = kx_1$:

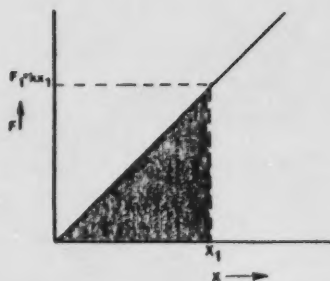


FIG. 3

$$A = \frac{1}{2} bh$$

Substitution yields the following:

$$W_1 = \frac{1}{2} (x_1)(kx_1)$$

$$W_1 = \frac{1}{2} kx_1^2$$

The potential energy of a spring is equal to work done on the spring and is given by Equation 3.

$$E_p = \frac{1}{2} kx^2 \quad (3)$$

where

E_p = potential energy.

Example B demonstrates the use of Equation 3 in finding the potential energy of a spring.

EXAMPLE B: POTENTIAL ENERGY STORED IN A SPRING

Given: A mass of 2 kg is suspended from a spring and stretches the spring by 10 cm.

Find: Potential energy stored in the spring.

Solution: Determine spring constant from Equation 2.

$$\begin{aligned} F &= mg \\ &= (2 \text{ kg})(-9.8 \text{ m/s}^2) \\ F &= -19.6 \text{ N (choose down as positive)} \\ x &= 0.1 \text{ m} \\ k &= -\frac{F}{x} \quad (\text{Equation 2}) \\ &= -\frac{(-19.6 \text{ N})}{0.1 \text{ m}} \\ k &= 196 \text{ N/m} \\ E_p &= \frac{1}{2} kx^2 \quad (\text{Equation 3}) \\ &= (0.5)(196 \text{ N/m})(0.1 \text{ m})^2 \\ &= 0.98 \text{ N}\cdot\text{m} \\ E_p &= 0.98 \text{ J (Recall: 1 joule = 1 newton-meter)} \end{aligned}$$

POTENTIAL AND KINETIC ENERGY OF A SPRING-MASS OSCILLATOR

Figure 4 displays an ideal spring-mass oscillator. A mass m is attached to one end of a spring having spring constant k . The other end is attached to a fixed vertical support. All frictional forces are neglected in the following discussion.

If the spring is stretched to displacement x_1 and released, the restoring force will pull the mass back toward the equilibrium position x_0 . The potential energy of the spring is converted to kinetic energy of the mass, and it moves past the equilibrium position until this

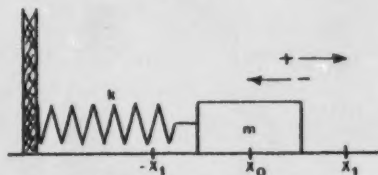


FIG. 4

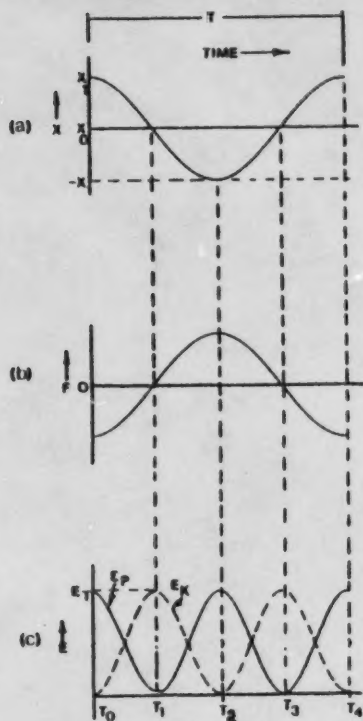


FIG 5

In Equation 4, the spring's mass is considered negligible by comparison to the mass attached to it. This equation is clarified in Example C.

kinetic energy once again is stored as potential energy in the spring at point $-x_1$. The result is an oscillatory motion, described graphically in Figure 5.

Figure 5a shows displacement as a function of time. Figure 5b shows the corresponding force produced by the spring. Figure 5c indicates both potential energy stored in the spring and kinetic energy of the mass. The sum of these two terms always is equal to E_t , the total energy of the system. At times t_0 and t_4 , the spring is stretched to x_1 , and all the energy is potential energy in the spring. At time t_2 , the spring is compressed to $-x_1$, and all the energy is potential energy. At these times, the velocity of the mass is zero, giving no kinetic energy. At times t_1 and t_3 , all the energy is kinetic energy. At these times, the displacement is zero, and the velocity is maximum.

The oscillatory motion of the spring occurs with a period T given by Equation 4.

$$T = 2\pi\sqrt{\frac{m}{k}} \quad (4)$$

where

T = period of oscillatory spring motion.

m = mass attached to spring.

EXAMPLE C: OSCILLATION OF A SPRING-MASS SYSTEM

Given: In Figure 5, $k = 200 \text{ N/m}$, $m = 1 \text{ kg}$, and $x_1 = 5 \text{ cm}$.

Find: (a) Period of spring oscillation.
(b) Total energy of the system.
(c) Maximum velocity of the mass.

Solution: (a) $T = 2\pi\sqrt{\frac{m}{k}}$

$$\begin{aligned}
 T &= (2)(3.14)\sqrt{\frac{1 \text{ kg}}{200 \text{ kg/s}^2}} \quad (200 \text{ N/m} = 200 \text{ kg/s}^2) \\
 &= (6.28)(0.0707 \text{ s}) \\
 T &= 0.44 \text{ s}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad E_p &= \frac{1}{2} kx^2 \\
 &= (0.5)(200 \text{ N/m})(0.05 \text{ m})^2 \\
 E_p &= 0.25 \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad E_K &= \frac{1}{2} mV^2 \\
 V &= \sqrt{\frac{2 E_K}{m}} \\
 &= \sqrt{\frac{2 (0.25 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2})}{(1 \text{ kg})}} \\
 &= \sqrt{0.5 \text{ m}^2/\text{s}^2} \\
 V &= 0.707 \text{ m/s}
 \end{aligned}$$

In a real spring-mass system, friction acts as a "damping force" that damps out the oscillations by converting the energy of the system to heat energy. The oscillations die out, as indicated by Figure 6.

APPLICATIONS OF SPRINGS

In all cases, springs act as energy-storage devices, as illustrated in the following examples:

- The rubber band in a sling-shot converts the potential energy of a stretched sling into the kinetic energy of the projectile. A converse example applied on a large scale is the strong rubber cord used on an aircraft carrier to stop the landing airplane by converting the kinetic energy of a landing plane into the potential energy of a stretched cord.
- The energy necessary for the operation of a mechanical clock or watch is stored as potential energy in a coil spring.
- The balance wheel of a clock is a torsion pendulum composed of a rotating wheel and a torsion spring. The period of this oscillating energy system provides a constant time base for the clock.
- The coil springs or leaf springs of an automobile absorb the energy of sharp jolts and release this energy over a longer period of time, smoothing the ride.

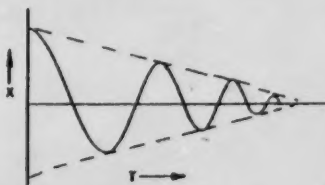


FIG. 6

SUMMARY

A spring is a device that stores mechanical energy. The force exerted by a spring is called a "restoring force" and always has a direction opposite that of the displacement. The spring constant describes the stiffness of the spring in terms of the force necessary to produce a given displacement. A spring and a mass form an oscillating energy system, which, under conditions of negligible friction, has a fixed period that can be used for time measurements.

LABORATORY

MEASUREMENT OF SPRING CONSTANT

MATERIALS

Hooke's Law apparatus (Sargent-Welch #0569 or equivalent) consisting of stand, spring, displacement scale, weight hanger, slotted weights (Figure 7).
Stop watch.

PROCEDURE

1. Assemble the equipment illustrated in Figure 7.
2. Record position of indicator with no mass on weight hanger.
3. Add weights to hanger. Record total mass added and new position of indicator.
4. Calculate displacement, spring force, and spring constant; record in Data Table.
5. Pull mass downward to a displacement of twice that measured in Step 4. Release mass and record time for 10 oscillations of spring-mass system.
6. Calculate period of oscillations and spring constant k . (Use Equation 4 and total mass of weights and hanger.)
7. How would a mass twice as large affect the results, and why?

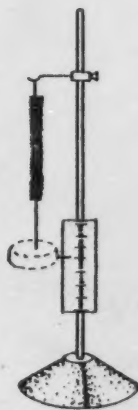


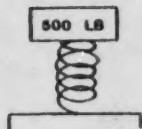
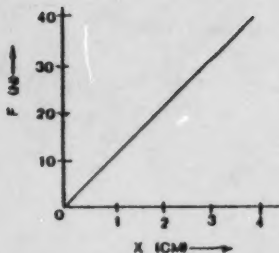
FIG. 7

DATA TABLE	
Initial position of indicator:	$h_1 =$ _____
Final position of indicator:	$h_2 =$ _____
Mass added:	$m =$ _____
Displacement:	$x = h_1 - h_2 =$ _____
Spring force:	$F = mg =$ _____
Spring constant:	$k = \frac{mg}{x} =$ _____
Time for 10 oscillations:	$10 T =$ _____
Period:	$T =$ _____
Spring constant:	$k =$ _____

SELF CHECK

1. Define the following terms:

- Restoring force.
- Spring constant.
- Hooke's Law.



- From the adjoining graph, determine the energy stored in the spring when it is stretched 3 cm, and the spring constant.
- An unknown mass is suspended from a spring with a spring constant of 50 N/m. The spring is stretched by 4.2 cm. Determine the following:
 - Mass suspended from spring.
 - Potential energy stored.
- A coil spring supports a 500-lb weight and is compressed by 4 inches. Determine the following:
 - Spring constant.
 - Stored energy.
 - Period of oscillation of this system.
- More weight is added to the spring in Problem 4 to increase its compression to 8 inches. Determine the following:
 - Weight added.
 - Stored energy.
 - Period of oscillation of new system.
- Draw diagrams showing the following quantities as functions of time for an oscillating spring-mass system:
 - Displacement.
 - Potential energy stored in spring.
 - Kinetic energy of mass.
- Explain how energy is stored in a spring in each of the following applications:
 - Wind-up toy.
 - Crossbow.
 - Automobile suspension.

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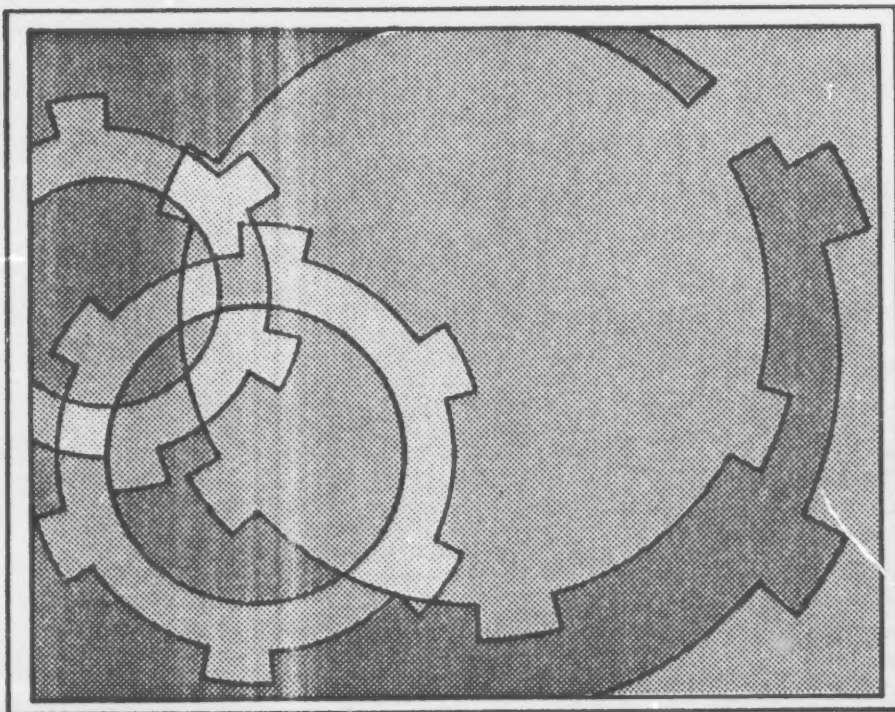
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UNIFIED TECHNICAL CONCEPTS

MODULE 7 M 3

ROTATIONAL KINETIC ENERGY
OF A FLYWHEEL



MECHANICAL APPLICATION OF POTENTIAL
AND KINETIC ENERGY

Technical Education Research Center-Southwest

TERC - SW

INTRODUCTION

A flywheel is a device used to store mechanical energy in the form of rotational motion of a mass. This rotational kinetic energy can be converted to other forms for use at a later time.

In this module, the student will examine characteristics of flywheels as energy-storage devices. In the laboratory, the student will construct a simple flywheel and will determine its efficiency as an energy-storage device.

PREREQUISITES

Student should have completed Concept Module 7-0, "Potential and Kinetic Energy."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Given mass and diameter of a uniform disk, calculate its moment of inertia.
- Calculate kinetic energy of a planetary body in a circular orbit, given its mass, angular velocity and radius of orbit.
- Calculate kinetic energy stored in a flywheel, given its moment of inertia and angular velocity.
- Calculate angular velocity of a flywheel, given its kinetic energy and moment of inertia.
- Describe energy losses that occur in a simple flywheel system.
- Given the appropriate equipment, construct a simple flywheel and determine its stored energy and angular velocity.

DISCUSSION

Figure 1 shows a particle of mass m in a circular orbit of radius r . Velocity of the particle is v , and angular velocity is $\omega = v/r$, where r is the radius.

In Concept Module 7-0, "Potential and Kinetic Energy," kinetic energy of such a mass was shown to be the following:

$$E_k = \frac{1}{2} mv^2 = \frac{1}{2} mr^2 \omega^2$$

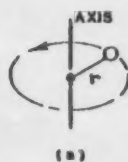
From Figure 2, the moment of inertia of a particle in a circular orbit is

$$I = mr^2.$$

Thus, kinetic energy of the orbital mass is given by Equation 1.

$$E_k = \frac{1}{2} I \omega^2$$

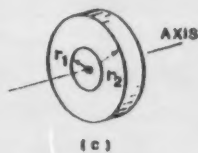
Equation 1 gives the kinetic energy of any rotating mass, as long as the correct expression is substituted for I , moment of inertia. In every case, the proper moment of inertia is that about the axis around which the body is rotating. Examples A and B show use of Equation 1 in solving problems.



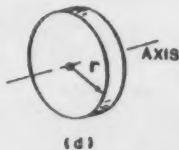
(a)
Particle in circular orbit.
 $I = mr^2$



(b)
Slender rod, axis through center.
 $I = \frac{1}{12} ml^2$



(c)
Annular (ring) cylinder.
 $I = \frac{1}{2} m (r_1^2 + r_2^2)$



(d)
Solid cylinder.
 $I = \frac{1}{2} mr^2$



(e)
Thin-walled hollow cylinder (wheel).
 $I = mr^2$



(f)
Solid sphere.
 $I = \frac{2}{5} mr^2$

FIG. 2

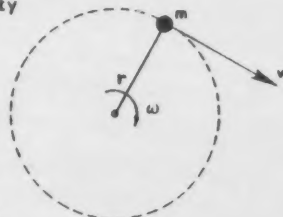


FIG. 1

EXAMPLE A: ROTATIONAL K E OF A THIN-WALLED HOLLOW CYLINDER

Given: A thin-walled cylinder with a mass of 15 kg and a radius of 18 cm spinning at 2000 rpm.

Find: Its rotational kinetic energy.

Solution: $I = mr^2$ (from Figure 2)

$$= (15 \text{ kg})(0.18 \text{ m})^2$$

$$I = 0.486 \text{ kg} \times \text{m}^2$$

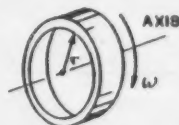
$$E_k = \frac{1}{2} I \omega^2$$

$$= (0.5)(0.486 \text{ kg} \times \text{m}^2)(209 \frac{\text{rad}}{\text{s}})^2$$

$$E_k = 1.06 \times 10^4 \text{ J}$$

$$\omega = 2000 \frac{\text{r}}{\text{min}} \left(\frac{2\pi \text{ rad}}{1 \text{ r}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right)$$

$$\omega = 209 \frac{\text{rad}}{\text{s}}$$



EXAMPLE B: ROTATIONAL RATE OF A SOLID CYLINDER

Given: A solid cylinder with a mass of 25 kg and a radius of 25 cm used to store kinetic energy.

Find: What its rotational rate must be in rpm to store 100 kJ.

Solution:

$$I = \frac{1}{2} mr^2 \text{ (from Figure 2)}$$

$$= (0.5)(25 \text{ kg})(0.25 \text{ m})^2$$

$$I = 0.781 \text{ kg} \times \text{m}^2$$

$$\text{rpm} = (506 \frac{\text{rad}}{\text{s}}) \left(\frac{1 \text{ r}}{2\pi \text{ rad}} \right) \left(\frac{60 \text{ s}}{1 \text{ min}} \right)$$

$$\text{rpm} = 4832 \text{ rpm}$$

$$E_k = \frac{1}{2} I \omega^2$$

$$\omega = \sqrt{\frac{2 E_k}{I}}$$

$$= \sqrt{\frac{2 (10^5 \text{ J})}{0.781 \text{ kg} \times \text{m}^2}}$$

$$\omega = 506 \text{ rad/s}$$



APPLICATIONS OF FLYWHEELS

In many mechanical devices, flywheels are used to smooth the motion of reciprocating parts. Figure 3 shows the flywheel and one piston of an 8-cylinder internal-combustion engine. In this system part of the kinetic energy of the flywheel is used to force the piston upward to compress the air-fuel mixture in the cylinder. When the mixture burns, some of the energy produced is stored in the flywheel to be used in the next compression stroke.

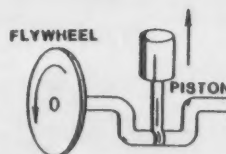


FIG. 3

The friction motors of toy cars are excellent examples of pure kinetic energy drives. The axle is connected to a flywheel by a series of gears. When the toy is pushed forward, the flywheel is caused to spin at a high rate. When the toy is released, the stored kinetic energy is transmitted to the axle by the same gear system, to maintain the forward motion.

In recent years, several investigations have considered storage of large amounts of rotational kinetic energy in flywheels as a solution to practical transportation problems. Flywheels weighing as much as 3000 lb, and smaller wheels with rotational rates of as high as 12,000 rpm, have been installed in vehicles. In some systems, direct mechanical coupling of the flywheel to the drive wheels has been employed; but most use the electrical conversion system shown in Figure 4. In this system, the flywheel drives a generator that supplies electrical power to a motor that powers the vehicle.

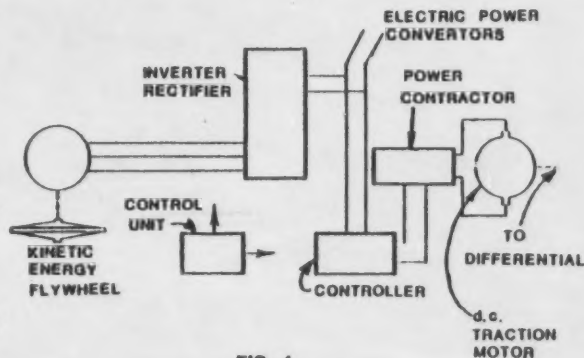


FIG. 4

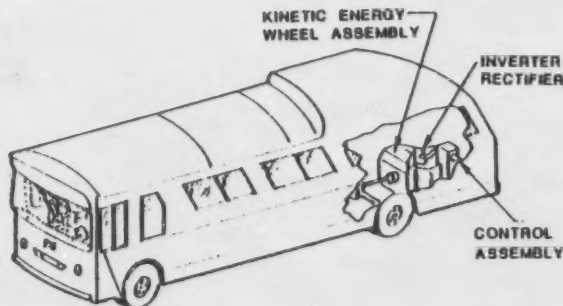


FIG 5

Figure 5 shows a bus equipped with a kinetic-energy drive. This drive stores 25 MJ, and gives the bus a range of several miles. A similar system can run a small automobile for 7 to 9 miles at speeds of 30 miles per hour. Combinations of

kinetic energy and electrical drives offer both lower energy utilization and lower pollution for urban transportation.

SUMMARY

A turning flywheel possesses kinetic energy in the form of the mechanical, rotational energy of its mass. Kinetic energy stored in flywheels is used to smooth the motion of reciprocating parts (as, for example, in engines) and to provide energy for drive systems in some vehicles.

LABORATORY

ROTATIONAL KINETIC ENERGY

MATERIALS

Wheel and axle. The wheel is an 18-inch circular disk of 1/2-inch aluminum. It is mounted on an axle consisting of 1-inch diameter wooden dowel.

Two 1-inch bearings.

Tack.

Nylon twine.

Metric weights.

Meter stick.

Metric balance.

PROCEDURE

1. Weigh wheel and axle; record appropriate mass (kilograms) in Data Table. Record radius of wheel; calculate its moment of inertia. (Note: This calculation is based upon a solid, uniform disk, and ignores the small change due to presence of wooden axle at center of disk.)
2. Mount wheel as shown in Figure 6. Be sure that wheel turns freely. Fasten string securely to axle by pressing a tack through end to prevent slipping. Cut other end of string at a length that will allow weight to hang without touching table top. Attach 200 g to free end of string. Measure, and record distance (h_1) from bottom of weight to table top in Data Table.
3. Turn wheel to wind string around axle, raising weight almost to axle. Measure, and record distance (h_2) from bottom of weight to table top in Data Table.
4. Release wheel, and allow weight to drop, storing kinetic energy in flywheel. The stored energy then will raise weight above table as string is rewound on axle. Hold wheel when

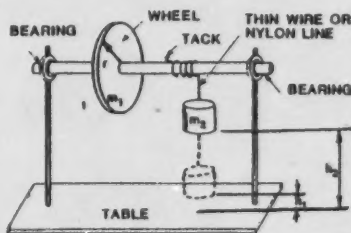


FIG. 6

It comes to a stop. Measure, and record new height (h_3) of weight in Data Table.

5. Complete calculations in Data Table.
6. Repeat with a 500-g weight.
7. What happened to energy lost in this flywheel system?

DATA TABLE
Trial 1 Mass of wheel and axle: $m_1 = \underline{\hspace{2cm}}$ kg Radius of wheel: $r = \underline{\hspace{2cm}}$ m Moment of inertia: $I = \frac{1}{2} m r^2$ $= 0.5 \times (\underline{\hspace{2cm}} \text{ kg}) (\underline{\hspace{2cm}} \text{ m})^2$ $I = \underline{\hspace{2cm}} \text{ kg} \times \text{m}^2$ Suspended mass = $m_2 = \underline{\hspace{2cm}}$ kg Minimum height of suspended weight: $h_1 = \underline{\hspace{2cm}}$ m Height of weight before release: $h_2 = \underline{\hspace{2cm}}$ m Height of weight after return: $h_3 = \underline{\hspace{2cm}}$ m Distance weight fell: $d_1 = h_2 - h_1 = (\underline{\hspace{2cm}} \text{ m}) - (\underline{\hspace{2cm}} \text{ m})$ $= \underline{\hspace{2cm}} \text{ m}$ Distance weight rose: $d_2 = h_3 - h_1 = (\underline{\hspace{2cm}} \text{ m}) - (\underline{\hspace{2cm}} \text{ m})$ $= \underline{\hspace{2cm}} \text{ m}$ Potential energy of weight before release: $E_{p1} = m_2 d_1 g = \underline{\hspace{2cm}} \text{ kg} \times \underline{\hspace{2cm}} \text{ m} \times 9.8 \text{ m/s}^2 = \underline{\hspace{2cm}} \text{ J}$ Potential energy of weight after return: $E_{p2} = m_2 d_2 g = \underline{\hspace{2cm}} \text{ kg} \times \underline{\hspace{2cm}} \text{ m} \times 9.8 \text{ m/s}^2 = \underline{\hspace{2cm}} \text{ J}$ % Efficiency of energy storage: $\eta = \frac{E_{p1} - E_{p2}}{E_{p1}} \times 100 = \frac{(\underline{\hspace{2cm}} \text{ J}) - (\underline{\hspace{2cm}} \text{ J})}{(\underline{\hspace{2cm}} \text{ J})} \times 100$ $= \underline{\hspace{2cm}} \%$ Assume that half of the energy loss occurs during conversion of potential energy of weight to kinetic energy of flywheel. Maximum kinetic energy of the flywheel then is as follows:

$$\text{Stored energy: } E = \frac{E_{p1} + E_{p2}}{2} = 0.5 \times (\text{ } J + \text{ } J) \\ = \text{ } J$$

$$\text{Maximum angular velocity of flywheel: } E = \frac{1}{2} I \omega^2$$

$$\omega = \sqrt{\frac{2 E}{I}}$$

$$= \sqrt{\frac{2 \times (\text{ } J)}{(\text{ } \text{kg} \times \text{m}^2)}}$$

$$\omega = \text{ } \text{rad/s}$$

Trial 2

$$I = \text{ } \text{kg} \times \text{m}^2$$

$$m_2 = \text{ } \text{kg}$$

$$d_1 = \text{ } \text{m}$$

$$d_2 = \text{ } \text{m}$$

$$E_{p1} = \text{ } J$$

$$E_{p2} = \text{ } J$$

$$\eta (\% \text{ eff}) = \text{ } \%$$

$$\text{Stored Energy: } E = \text{ } J$$

$$\omega_{\text{max}} = \text{ } \text{rad/s}$$

SELF CHECK

1. A uniform disk of mass 4.7 kg has a diameter of 35 cm. Calculate its moment of inertia.
2. A body of mass 2.2 kg moves in a circular orbit with a radius of 0.85 m and angular velocity of 50 rad/s. What is its kinetic energy?
3. A flywheel consists of a uniform disk with a radius of 30 cm and a mass of 400 kg. If it spins at 5000 rpm, what is the stored kinetic energy?
4. Assume that a flywheel has a kinetic energy of 10 kJ and a moment of inertia of 0.032 kg x m². Calculate its angular velocity.

5. At what rate must the above flywheel spin to store 2 MJ?

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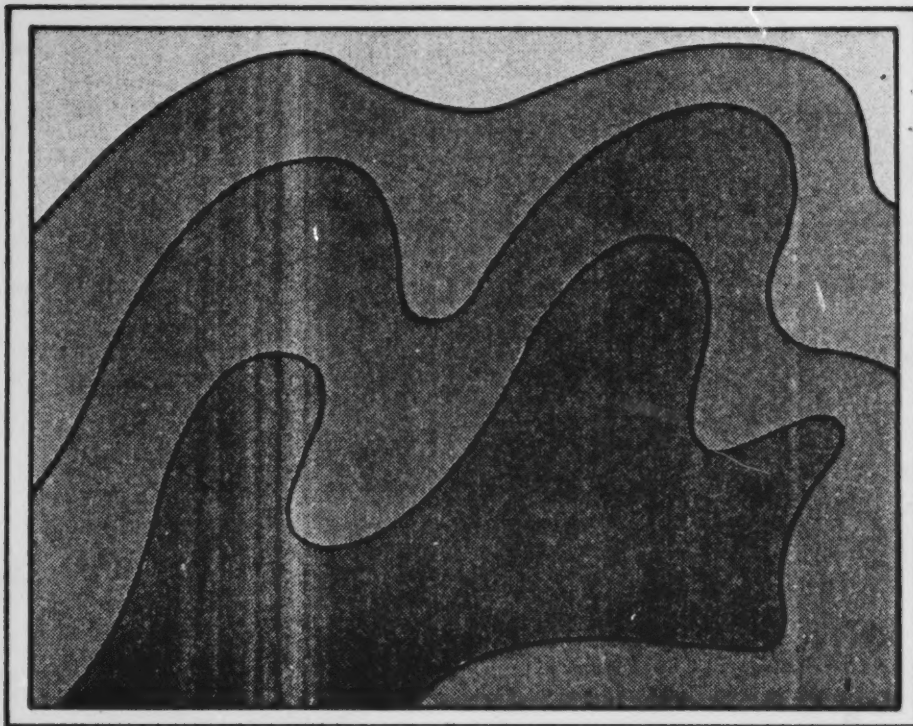
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UNIFIED TECHNICAL CONCEPTS

MODULE 7 F 1

ENERGY STORED
IN LARGE TANKS



FLUID APPLICATION OF
POTENTIAL AND KINETIC ENERGY

Technical Education Research Center - Southwest

TEC - SW

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INTRODUCTION

Torricelli's theorem is a special case of Bernoulli's equation, discussed in Concept Module 7-0. It is applicable in situations of large reservoirs of liquids having outlets some distance below the surface of the liquid. Fluid kinetic energy at the outlet is provided only by the gravitational potential energy stored in the liquid above the level of the outlet. Torricelli's theorem enables a velocity of flow to be theoretically predicted as a simple function of the height of the fluid level in the reservoir.

This module will present the development of Torricelli's theorem from Bernoulli's equation. A discussion of the energy exchanges (kinetic and potential energies) that occur in steady nonviscous incompressible fluid flow is included, as well as specific conditions for which Torricelli's theorem is valid. An experiment is included to provide a laboratory experience in which the conditions of Torricelli's theorem are simulated, thereby illustrating a test of the validity of the theorem.

PREREQUISITES

Student should have completed Concept Module 7-0, "Kinetic and Potential Energy."

OBJECTIVES

To complete this module, the student should be able to:

- Describe conditions and assumptions upon which Torricelli's theorem is derived from Bernoulli's equation.
- Identify terms in Bernoulli's equation associated with potential energy, gravitational potential energy and kinetic energy of a fluid.
- Substitute metric units into Bernoulli's equation to show dimensional consistency.

- Perform various calculations using Torricelli's theorem to find flow and velocity.
- Show that an alternate (mass-less) method of determining velocity of a fluid can be achieved.
- Calculate fluid velocity for an outlet pipe of a large reservoir of fluid (Torricelli's theorem).

DISCUSSION

In Concept Module 7-0, Bernoulli's equation was explained as a law of conservation of energy that applies to streamline flow of an incompressible fluid. Bernoulli's equation, simply stated in words, reminds us that net work done on a fluid or by a fluid must be equal to the change in energy of the fluid mass. Net work done on or by the fluid must result from work equal to the change of the fluid's kinetic energy and the work equal to a change in the gravitational potential energy. This energy relationship is illustrated in Equation 1 in the form in which Bernoulli's relationship is usually seen.

$$(P + \rho Gh + \frac{1}{2} \rho v^2) = \text{constant} \quad (1)$$

Term for fluid pressure. Term for gravitational potential energy. Term for kinetic energy.

where

P = pressure of fluid at a given point.

ρ = density of fluid (a constant term for incompressible fluid).

G = acceleration due to gravity (a constant term).

h = height of fluid mass at the given point above a reference height.

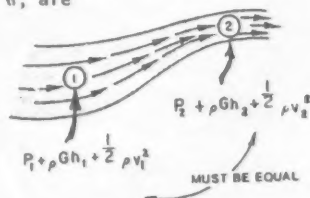
v = velocity of fluid at given point in the stream.

A close examination of Equation 1 will reveal that the variables necessary to be measured at a given point in the flow are pressure, height and velocity. The other terms, ρ and G , are constants.

The sum of the terms on the left side of Equation 1 must be the same (constant) at any (and all) points along the streamline flow as shown in Figure 1. Frequently the sums of these terms are compared at two separate points, resulting in Equation 2.

$$P_1 + \rho Gh_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho Gh_2 + \frac{1}{2} \rho v_2^2.$$

Equation 2 forms the basis of the special case called Torricelli's law (or theorem).



(2) Fig. 1

TORRICELLI'S THEOREM

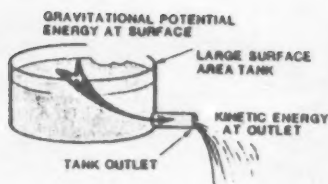


Fig. 2

Torricelli's theorem is a special application of Bernoulli's conservation of energy equation. The theorem is based upon the special case that occurs when gravitational potential energy of a fluid at a large reservoir surface level is converted to fluid kinetic energy at an outlet below surface level of the reservoir, as shown in Figure 2.

For Torricelli's theorem to be valid, all conditions and assumptions for Bernoulli's equation must be met, as well as certain conditions and assumptions unique to the application. These conditions will be explained later in this module.

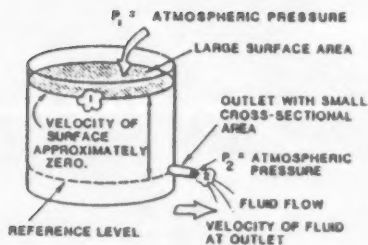


Fig. 3

Figure 3 shows a large container of fluid with an outlet near the bottom and open to the atmosphere at the top. Intuition will predict the result when a hole is punctured in a container open to the atmosphere. The surface of the liquid in the container will drop with a decreasing speed. However, if an olympic-size swimming pool was being drained through a 1/4 inch diameter pipe, the fluid surface of the swimming pool would drop slowly and its velocity would be extremely small. If this reasoning is carried further, for a reservoir of infinite surface area, velocity of the surface would be zero. This is the condition for which Bernoulli's equation will result in Torricelli's law.

Also, both surface and outlet must be subject to atmospheric pressure (or at least the same pressure).

If conditions stated and illustrated in Figure 3 are applied to Equation 2, the result will be an expression of Torricelli's law.

$$P_1 + \rho gh_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2} \rho v_2^2$$

At the surface of reservoir.

At the outlet tube.

Substitute the conditions:

$P_1 = P_2$ (atmospheric pressure).

$h_2 = 0$ (reference level at outlet tube).

$v_1 = 0$ (velocity at the reservoir surface).

$$\cancel{(\text{Atmospheric pressure})} + \rho G h_1 + \frac{1}{2} \rho (0)^2 = \cancel{(\text{atmospheric pressure})} + \rho G (0) + \frac{1}{2} \rho v_2^2$$

Gravitational potential energy at surface.

$$\rho G h_1 = \frac{1}{2} \rho v_2^2$$

Kinetic energy of fluid at outlet.

Mathematical statement of Torricelli's law.

$$\rightarrow \sqrt{2 G h_1} = v_2.$$

(3)

In summary, gravitational potential energy stored in the fluid of height h_1 is released in the form of kinetic energy of the fluid mass at the outlet.

To determine velocity of the fluid draining a large reservoir under the action of gravity alone, use Equation 3. A sample problem is shown in Example A.

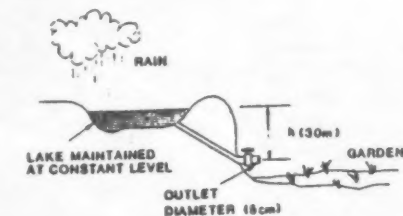
The laboratory experiment in this module simulates the conditions of a constant reservoir surface level by maintaining a flow of water into the finite size container such that fluid surface level in the container has a "zero" velocity.

EXAMPLE A: VELOCITY OF OUTLET FLUID FROM LARGE RESERVOIR

Given: A farmer wishes to irrigate a garden by draining a large lake. The drain pipe outlet is 5 cm inch diameter, and the outlet is a distance (h) of 30 meters below the surface of the lake.

find: How many cubic meters of water per hour could be used to water the garden.

Solution: (a) Use Torricelli's law to find velocity of the fluid at the outlet:



$$v = \sqrt{2 Gh}$$

$$= \sqrt{2 \times 9.8 \frac{\text{m}}{\text{s}^2} \times 30 \text{ m}}$$

$$v = 24.25 \frac{\text{m}}{\text{s}}$$

(b) Volume flow rate = (cross-sectional area of outlet) $\times$ (fluid velocity)

$$= \left[\frac{\pi}{4} \times (0.05 \text{ m})^2 \right] \times 24.25 \frac{\text{m}}{\text{s}}$$

$$= 0.0476 \frac{\text{m}^3}{\text{s}} \times \frac{3600 \text{ s}}{\text{h}}$$

$$\text{Volume flow rate} = 171 \frac{\text{m}^3}{\text{h}}$$

LABORATORY

DEMONSTRATION OF TORRICELLI'S THEOREM

MATERIALS

Aspirator bottle, large capacity (8 liters) clear glass with exit tube suitable for 1/4 inch hose. (Available from scientific supply company. A large container with a hole at the bottom may be substituted; however, the aspirator bottle outlet is shaped for minimum frictional loss, thereby achieving closer agreement with theory.)

Meter stick or metric tape measure.

Vernier caliper with inside diameter measuring capability.

Two large containers (5-gallon buckets).

Two platform scales graduated in kilograms.

Stopwatch.

Rubber stop (size to fit outlet).

Adjustable water supply.

Optional: Movable platform constructed of 3/4-inch plywood and supported by inexpensive heavy appliance rollers.

PROCEDURE

1. Measure inside diameter of exit tube of aspirator bottle with vernier caliper; record this diameter in data table.
2. Arrange aspirator bottle, collection buckets and associated equipment as shown in Figure 4.
3. Weigh collection container (empty and dry); record weight in data table.
4. Plug exit tube with a rubber stopper or stop flow manually (with thumb or finger). Turn faucet on to fill container to desired height ("h" in Figure 4 to approximately

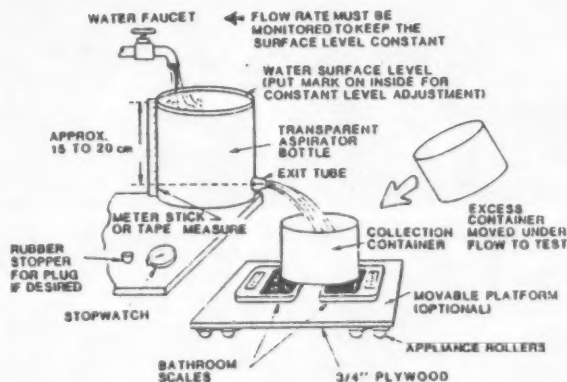


Fig. 4

- 15-20 centimeters).
5. Position excess container underneath expected flow direction; release plug at exit tube. Faucet flow must be adjusted to correct flow rate so that level in container will remain constant. (Some practice will show that this is not a difficult task, especially if a mark is put on inside of container.)
 6. Raise excess container, placing collection container and scales underneath. Remove excess container from stream of flow while simultaneously starting stopwatch.
 7. After a large amount of water is collected in collection container, stop the stopwatch as excess container is simultaneously moved underneath flow. (Preplanning of this step will aid in reducing errors in total water amount.) Shut off faucet; plug exit tube.
 8. Record time interval and mass of water collected during trial in data table.
 9. Empty collection and excess containers; reset stopwatch.
 10. Repeat steps 4, 5, 6, 7, 8 and 9 for three additional heights, ranging from 15 cm to 2 cm above reference level of exit tube. Record each height and associated trial data in data table.
 11. Complete calculations necessary in data table:
 - a. Calculate inside cross-sectional area (A) of exit tube; record in table.
 - b. Calculate volume flow rate by the following equation:

$$\text{Volume flow rate} = \frac{\text{Mass flow rate}}{\text{Density of water}} = \frac{\left(\frac{\text{mass collected in trial}}{\text{time of trial}} \right)}{(1000 \text{ kg/m}^3)}$$

Record volume flow rate for each trial.

- c. Divide each volume flow rate by cross-sectional area of exit tube to obtain experimental velocity of water flow through exit tube. Record this velocity for each trial.
- d. Use Torricelli's theorem to calculate velocity of water flow through exit tube:

$$v_{\text{calc}} \text{ (m/s)} = \sqrt{2 \left(9.80 \frac{\text{m}}{\text{s}^2} \right) h \text{ (m)}}$$

- e. Record V_{calc} for each trial in data table. Compare V_{calc} with V_{exp} for each trial, determine a percentage error.

DATA TABLE							
Trial	Height of Water Surface Level Above Reference Level (m)	Collection Time (s)	Mass of Water Collected* (kg)	Volume Flow Rate (m^3/s)	Experimental Velocity Of Flow At Exit Tube (m/s) V_{exp}	Calculated Velocity Of Flow At Exit Tube ($v = \sqrt{2 Gh}$) V_{calc}	% Error
1							
2							
3							
4							

Mass of empty collection container _____ kg.

Inside diameter of exit tube _____ m.

Circular cross-sectional area of exit tube _____ m^2 [area = $\frac{1}{4} \pi$ (diameter) ^{2}].

*Mass of water collected = (mass of water + container) - mass of empty container.

SELF CHECK

- Describe conditions and assumptions upon which Torricelli's theorem is derived from Bernoulli's equation.
- Identify terms in Bernoulli's equation associated with potential energy, gravitational potential energy and kinetic energy of a fluid.
- Show that Equation 1 is dimensionally consistent by substituting metric units for each variable.
- Would velocity at an actual outlet be equal to velocity pre-

dicted by Torricelli's theorem? State reasons for your answer.

5. The farmer in Example A changed piping so that a smooth irrigation pipe having a 10-cm diameter outlet drains water from the lake, 30 meters above the outlet. How many cubic meters of water per hour now flow onto the garden?
6. A large-diameter open top standpipe has water in it to a height of 1 m. At the bottom is a 1 cm² area hole. Calculate velocity of fluid exiting the hole. How much mass comes out of the hole per second?
7. Using kinematic equations of motion, show that velocity of fluid at the outlet of the reservoir (Figure 5) is described by the equation:

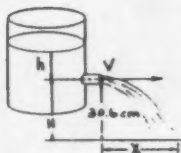


Fig. 5

$$v_{\text{outlet}} = \sqrt{\frac{G}{2H}} x \approx 4x \text{ m/s}$$

if x is measured in meters and $H = 30.6$ cm as shown in Figure 5. [This calculation suggests an alternate (massless) method of determining velocity of fluid at the outlet in the experiment of this module.]

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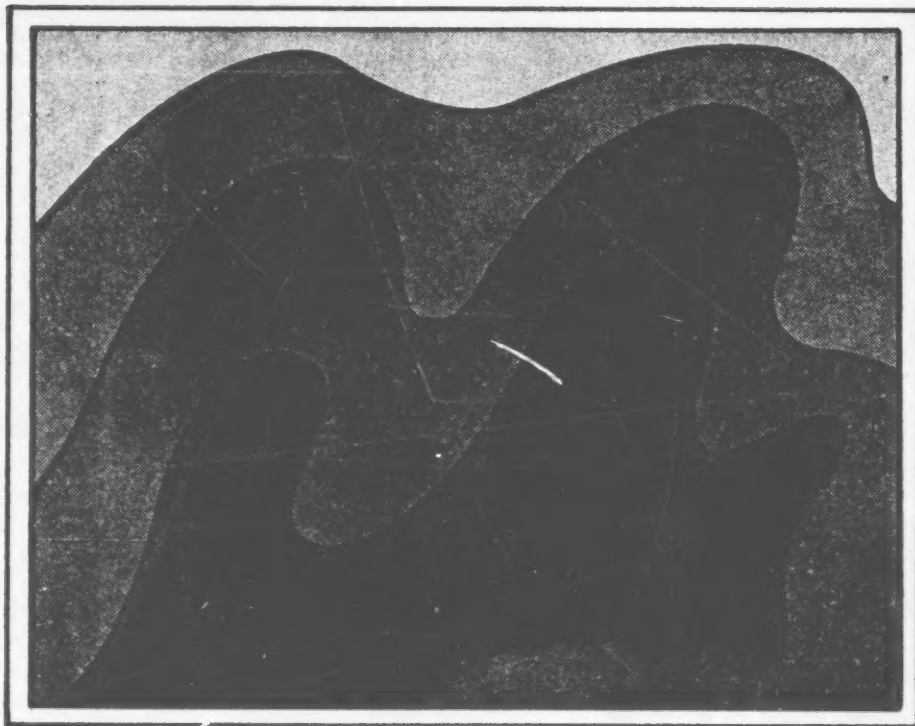
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UNIFIED TECHNICAL CONCEPTS

MODULE 7 F 2

VENTURI METERS



FLUID APPLICATION OF POTENTIAL AND KINETIC ENERGY

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INTRODUCTION

A venturi meter is a device used in industry for flow rate measurements. It illustrates the interchange of kinetic and potential energy that occurs in fluid flow. The energy interchange obeys the fluid conservation-of-energy equation, Bernoulli's theorem. Although Bernoulli's theorem is strictly correct only for incompressible, nonviscous liquids, it often is applied to all liquids with sufficient accuracy for many engineering purposes.

This module presents the application of Bernoulli's equation to a venturi meter and the derivation of an equation for the flow velocity in a horizontal pipe. A discussion of potential and kinetic energy changes for fluid flowing through a horizontal pipe is included. In the laboratory, the student will measure velocity of flow with a venturi meter.

PREREQUISITES

Student should have completed Concept Module 7-0, "Potential and Kinetic Energy."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Describe with the aid of diagrams and equations the application of Bernoulli's equation to fluid flow in a horizontal pipe in which the cross-sectional area changes along the flow direction.
- Calculate fluid velocity in a horizontal pipe using a velocity equation developed for a venturi flowmeter inserted into the pipe.
- Calculate pressure in the throat constriction of a venturi flowmeter.

- Given the appropriate equipment, measure the velocity of fluid flow with a venturi meter.

DISCUSSION

In Concept Module 7-0, Bernoulli's equation was explained as a conservation-of-energy law that applies to streamline flow of an incompressible fluid. Bernoulli's equation states that the net work done on, or by, a fluid must equal the change in kinetic and potential energy of the fluid mass. This energy relationship is given by Equation 1.

$$[p = \rho gh + \frac{1}{2} \rho v^2] = \text{constant} \quad (1)$$

where

- p = pressure of fluid at a given point.
- ρ = density of fluid (constant for incompressible fluid).
- g = acceleration due to gravity.
- h = height of fluid mass at given point above a reference level.
- v = velocity of fluid at given point in stream.

APPLICATION OF BERNOULLI'S EQUATION TO VENTURI METER

In this module, the changes in pressure and kinetic energy that occur at different cross sections in the venturi meter will be examined. This will permit determination of the flow velocity. To measure the flow rate of a liquid with a venturi flowmeter, a U-tube manometer can be attached to the bottom of the flowmeter, as illustrated in Figure 1.

The left-hand member of Equation 1 will be the same for any two points in 1 and 2 in Figure 1. Bernoulli's equation takes the form of Equation 2.

$$p_1 + \rho gh_1 + \frac{1}{2} \rho v_1^2 = p_2 + \rho gh_2 + \frac{1}{2} \rho v_2^2 \quad (2)$$

Because points 1 and 2 are the same elevation —

$$\rho gh_1 = \rho gh_2.$$

Equation 2 therefore reduces to the form of Equation 3 or 4.

$$p_1 + \frac{1}{2} \rho v_1^2 = p_2 + \frac{1}{2} \rho v_2^2 \quad (3)$$

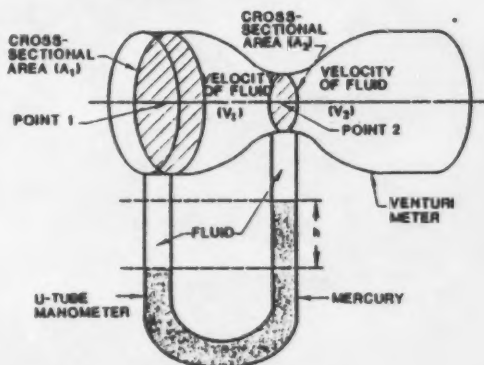
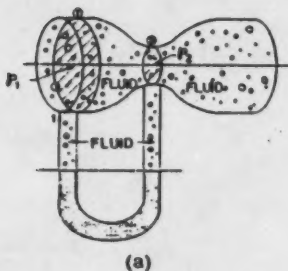


FIG. 1

or

$$p_1 - p_2 = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2 \quad (4)$$

In Equation 4, the pressure difference between points 1 and 2 (left-hand side) equals the change in kinetic energy of fluid flow between points 1 and 2 (right-hand side).



Pressure change between positions 1 and 2 in Figure 1 can be measured with a U-tube manometer. The simplified diagrams in Figure 2 illustrate how a U-tube manometer registers the pressure drop. In Figure 2a, representing a condition of no-flow in the venturi meter, the mercury in the U-tube is at the same level in each arm. In the lower drawing, under flow conditions, the pressure at position 2 decreases relative to position 1. This drop is registered by a difference in the height of the mercury level in the two arms of the U-tube.

Pressure difference $p_1 - p_2$ is obtained from Equation 5.

$$p_1 - p_2 = (\rho_H - \rho_F) gh \quad (5)$$

where

- ρ_H = density of mercury (13.6 g/cm³).
- ρ_F = density of liquid flowing through venturi flowmeter.
- h = height differential of mercury columns, cm.
- g = acceleration due to gravity (980 cm/s²).

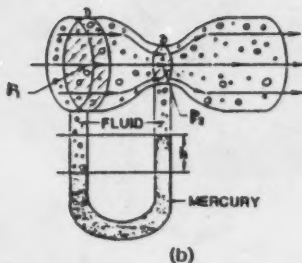


FIG. 2

For incompressible, streamline fluid flow, cross-sectional area and velocity are related in the manner shown by Equation 6.

$$A_1 v_1 = A_2 v_2 \quad (6)$$

where

- A_1 = cross-sectional area at position 1.
- v_1 = flow velocity at position 1.
- A_2 = cross-sectional area at position 2.
- v_2 = flow velocity at position 2.

The essential parameters p_1 , v_1 , A_1 ; and p_2 , v_2 , A_2 are shown in Figure 3 for positions 1 and 2.

Because the venturi flowmeter is inserted into a fluid system in which the natural flow velocity is v_1 , a single equation that yields v_1 in terms of known or measurable parameters is useful. When Equations 4, 5, and 6 are combined appropriately, Equation 7 is the result.

$$v_1 = \sqrt{2 \left(\frac{\rho_H - \rho_F}{\rho_F} \right) \left(\frac{A_2^2}{A_1^2 - A_2^2} \right) gh}$$

Flow velocity can be calculated from Equation 7 if the following values are known: density of mercury (ρ_H), density of the flowing liquid (ρ_F), cross-sectional area of the venturi flowmeter at the input (A_1) and at constriction (A_2), and the differential mercury height (h) in the U-tube manometer.

As the cross-sectional area is equal to $\pi d^2/4$ where d is diameter, Equation 7 can also be written in the form of Equation 8.

$$v_1 = \sqrt{2 \left(\frac{\rho_H - \rho_F}{\rho_F} \right) \left(\frac{d_1^4}{d_1^4 - d_2^4} \right) gh} \quad (8)$$

Example A illustrates use of Equation 8 in determining flow velocity in a flow pipe.

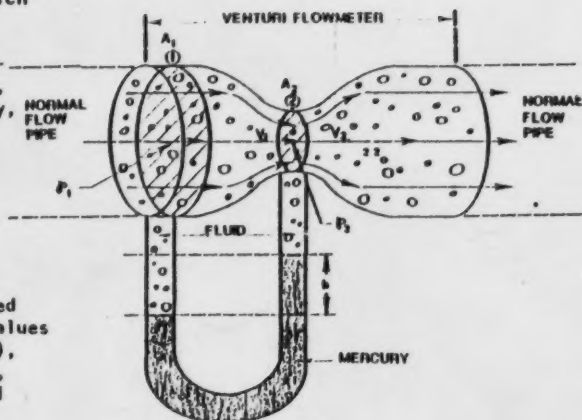
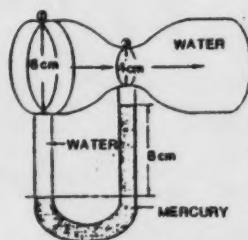


FIG. 3

EXAMPLE A: FLOW MEASUREMENT WITH A VENTURI FLOWMETER

Given: Water flows through a pipe 6-cm in diameter into a venturi tube with a constriction of 4 cm at the throat. Mercury fills the manometer U-tube; and, as the steady flow proceeds, a height differential of 8 cm is established.

Find: Velocity of fluid in the pipe.



Solution: From Equation 8 -

$$v_1 = \sqrt{2 \left(\frac{\rho_M - \rho_F}{\rho_F} \right) \left(\frac{d_2^4}{d_1^4 - d_2^4} \right) gh}$$

$$= \sqrt{2 \left(\frac{13.6 - 1}{1} \right) \left(\frac{(4)^4}{(6)^4 - (4)^4} \right) (980) (8)}$$

$$v = 220 \frac{\text{cm}}{\text{s}}$$

ENERGY LOSS IN A VENTURI TUBE

When fluid flows through a constriction in a real pipe or through a venturi flowmeter where resistance exists due to drag at the walls of the pipe, the constriction reduces total energy of the fluid. This reduction is analogous to the action of an electrical resistor that reduces electrical energy in a circuit and causes a voltage drop along the direction of current flow.

The experiment in this module uses a triple-arm manometer. The purpose of the third arm is to show that the fluid pressure recovered after flowing through the venturi flowmeter does not quite equal the pressure of the fluid at the entrance to the flowmeter. The third arm of the manometer therefore clearly shows the effect of frictional drag on pressure loss in real pipes. Any constriction or sudden bend in real hydraulic lines causes a similar pressure loss.

SUMMARY

The venturi tube is a short tube arranged with a constricted segment at the center to cause a pressure difference for the purpose of measuring the fluid flow in pipes. The fluid velocity is higher through the constriction than through either end of the tube. When fluid increases in velocity, it decreases in pressure, making possible a measurement of the rate of flow. Factors influencing measurement include the cross-sectional area of tube ends and restriction and the density of the fluid.

LABORATORY

USE OF A VENTURI METER TO DETERMINE FLOW VELOCITY

MATERIALS

Glass venturi tube (18 cm long, 18 mm in diameter; tube narrows at center and has three outlet tubes at center; ends of tube are flared). (Suggested sources: Science Kit Inc., Central Scientific Co., Sargeant Welch Co.)
Two lengths of 3/4-inch automobile heater hose.
Triple-arm manometer, 30 cm-long to fit outlet tubes of venturi tube. (Suggested source: Sargeant Welch Co.)
Rubber tubing (1/4-inch inside diameter) for connections to outlet tubes of venturi flowmeter.
Metric scales for manometer arms.
Circular clamp for 3/4-inch hose.
Two large containers (5-gallon buckets or trash cans).
Two platform scales graduated in kilograms.
Stopwatch.
Index cards (any size).
Scissors.

PROCEDURE

1. Arrange venturi tube, closed triple-arm mercury manometer, and associated equipment as shown in Figure 4.
2. Measure inside diameters of entrance and exits of venturi; record in Data Table 1.
3. Measure diameter of venturi throat, or obtain from manufacturer's specifications. Record in Data Table 1.
4. Place exit hose in excess water con-

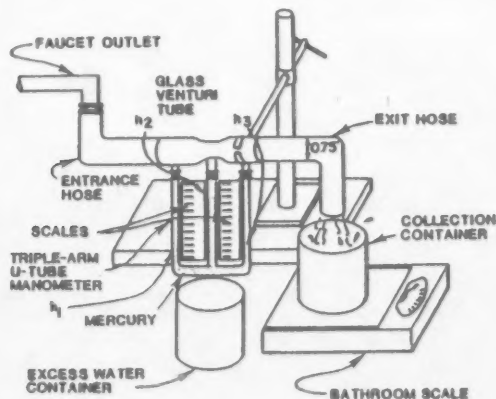


FIG. 4

tainer; slowly turn on faucet, checking system for leaks. Shut off faucet; repair leaks if necessary.

5. Slowly turn faucet on; adjust water flow to obtain a 2-cm difference in mercury levels of entrance and throat manometer arms.
6. Record mercury meniscus scale readings for all three arms of manometer.
7. Simultaneously switch exit hose to collection container, and start timer. After a time interval somewhat in excess of 2 minutes, stop timer, while simultaneously switching exit hose to excess water container. (Anticipate this action to minimize mess that can occur.)
8. Record total mass readings of scales and time elapsed (from timer).
9. Empty collection containers and excess water container as needed.
10. Repeat Steps 5, 6, 7, and 8 for mercury level differences of 6 and 10 cm between entrance (h_1) and venturi throat (h_2). Record data and results from each trial.
11. Do calculations necessary to complete Data Table 2.
 - a. Determine water added to collection container during each trial (kg at end of trial, minus kg at start of trial).
 - b. Calculate mass flow rate and volume flow rate. Calculate velocity v of fluid from relation given below. (This equation comes essentially from Equation 6 where the product of $A v$ is equal to a constant — the volume flow rate, Q_v .)

$$v = \frac{Q_v}{A_2}$$

Do this calculation for each trial in experiment. Record results in Data Table.

- c. For each trial, calculate velocity of fluid flow from Equation 8. Record results.
- d. For each trial, calculate a % difference between velocity in b and velocity in c. Record.
- e. For each trial, calculate pressure difference (Δp) between venturi entrance and throat from Equation 5. For measurements made in Figure 4, this becomes —

$$\Delta p = (\rho_H - \rho_F) g (h_2 - h_1)$$

- f. For each trial, calculate pressure difference (Δp) between venturi entrance and exit, using this equation:

$$\Delta p_v = (\rho_H - \rho_F) g (h_3 - h_1)$$

DATA TABLE 1. MEASUREMENTS						
Scale Readings of Mercury Meniscus Level				Scale Reading		
Trial	Entrance h_1 (cm)	Throat h_2 (cm)	Exit h_3 (cm)	Start of Trial (kg)	End of Trial (kg)	Water Added During Trial (kg)
1						
2						
3						
4						
5						

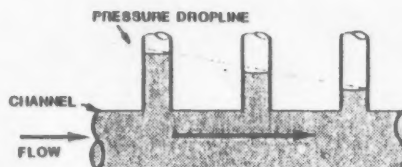
Inside diameters: Venturi entrance: _____ cm
 Venturi throat: _____ cm
 Venturi exit: _____ cm
 Time water is collected: _____ s

DATA TABLE 2. RESULTS OF CALCULATIONS

Trial	Mass Flow Rate (kg/s)	Volume Flow Rate (cm ³ /s)	Measured by Flow Rate (cm/s)	Velocity of Flow Calculated Using Equation 8 (cm/s)	% Difference	Pressure Change Δp (kg/m ²)	Pressure Loss Across Tube (Δp_v) (kg/m ²)
1							
2							
3							
4							
5							

SELF CHECK

1. Explain why height of liquid in the standpipes of the adjoining illustration indicates that pressure drops along the flow pipe, although the flow pipe has a uniform cross section and the flowing liquid is incompressible.
2. By a dimensional analysis, show that Bernoulli's equation is dimensionally consistent. Show that the units of each term in Bernoulli's equation (Equation 1) are energy per unit volume.
3. Height difference registered in the two arms of a mercury manometer with water above the mercury is 17.5 cm when connected to a horizontal venturi meter. Diameters of the pipe and throat are 15 cm and 7 cm, respectively. Neglecting frictional losses, calculate the volume rate of discharge through the meter.
4. A horizontal pipe has a cross-sectional area of 4.0 in^2 . In the center of the pipe, there is a constriction having a cross-sectional area of 1.0 in^2 . Gasoline (weight density 42 lb/ft^3) flows with a speed of 6.0 ft/s^2 in the larger section. Calculate (a) gasoline speed in the constriction and (b) pressure in the constriction.



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UTC

UNIFIED TECHNICAL CONCEPTS

MODULE 7 F 3

PITOT TUBES



FLUID APPLICATION OF POTENTIAL AND KINETIC ENERGY

Technical Education Research Center - Southwest

TEC - SW

INTRODUCTION

The pitot tube is a device designed for velocity measurement in a fluid. It finds extensive use on boats and aircraft as a simple, low-cost speed indicator. In measurement of air speeds and other fluid velocities in industrial applications, the pitot tube is used in an exploratory manner. It is used to check velocity of fluids passing through orifice plates, venturis, and other flow-measuring instruments, or to determine radial velocity distributions across a large pipe. Air or liquid impacting with the tube has kinetic energy that causes a measurable pressure differential.

This module will present the operating principles of the pitot tube, its advantages and its limitations in fluid velocity measurement, and its various features of construction and associated equipment. The pitot tube equation for velocity flow is derived from Bernoulli's equation.

In the laboratory, the student will measure the air speed of a moving automobile using a pitot tube.

PREREQUISITES

Student should have completed Concept Module 7-0, "Potential and Kinetic Energy."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Describe features of the construction of a pitot tube and environmental conditions in which it can be used.
- List merits and limitations of an industrial pitot tube given in this module.
- Use the pitot tube equation to calculate fluid speeds.
- Given the appropriate equipment, set up a pitot tube device in an actual experimental situation, and take measurements

necessary to calculate fluid flow velocity with the aid of the pitot-tube equation.

DISCUSSION

The pitot tube is a device that measures speed of fluid flow by measuring the impact pressure or force required to stop a small portion of the fluid. It is used for measurement of speed of a gas or of an incompressible liquid. A pitot tube of popular design is illustrated in Figure 1. It is constructed of two concentric tubes. The inner tube is open to the moving fluid, and is referred to as the "pitot tube." The outer tube, referred to as the "static tube," is shielded from the oncoming flow, although in its side it has small holes perpendicular to the flow of the fluid stream.

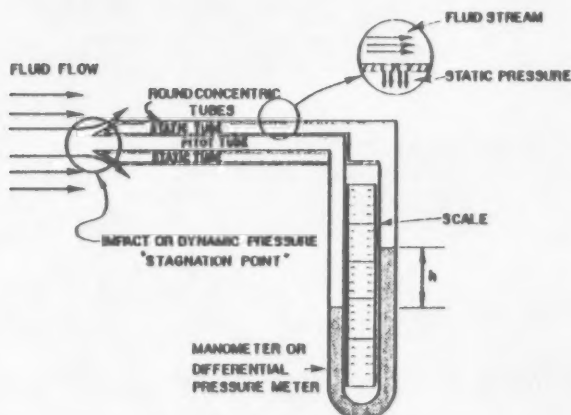


FIG. 1

The small holes in the static tube provide a means by which static stream pressure is measured. The dynamic, or impact, pressure occurs at the mouth of the instrument. Velocity of the fluid at this impact region is zero, and the region is sometimes called the "stagnation point." Speed of the fluid stream is proportional to the difference in impact pressure and static pressure. Pitot tubes use a sensitive differential-pressure measuring device, such as the manometer shown in Figure 1, to measure this pressure difference.

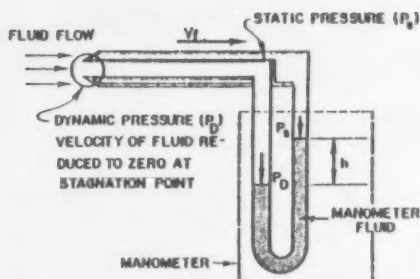


FIG. 2

The equation that describes the function of a pitot tube can be derived by applying conditions and variables illustrated in Figure 2 to Bernoulli's equation. The steps in this derivation provide a means of understanding the energy exchanges that occur in the pitot tube.

DERIVATION OF A PITOT TUBE EQUATION

Equation 1 consists of:

$$\begin{aligned} & \left(p_s + \frac{1}{2} \rho_F v_F^2 + \rho_F g h_s \right)_{\text{static}} \\ &= \left(p_d + \frac{1}{2} \rho_F v_d^2 + \rho_F g h_d \right)_{\text{dynamic}} \end{aligned} \quad (1)$$

where

- p_s = static pressure.
- ρ_F = density of fluid.
- v_F = velocity of fluid.
- g = acceleration due to gravity.
- p_d = dynamic pressure.
- v_d = dynamic velocity.
- h_s = elevation of static point.
- h_d = elevation of dynamic point.

At the stagnation point, velocity of the fluid v_d is zero. Elevations of the static and dynamic points are essentially equal, so the two terms cancel in Equation 1. When these conditions are applied to Equation 1, they yield the results shown in Equation 2.

$$\begin{aligned} & p_s + \frac{1}{2} \rho_F v_F^2 = p_d \\ \text{or} \quad & p_d - p_s = \frac{1}{2} \rho_F v_F^2 \end{aligned} \quad (2)$$

An examination of Equation 2 shows that the kinetic energy of the fluid impacting the central tube is absorbed and made evident in the form of a pressure differential.

Bernoulli's equation also can be applied to the manometric liquid, which is subject to the same pressure differential.

If h is difference in the height of the liquid in the manometric arm, and ρ_L is density of the manometric liquid, Bernoulli's equation would reduce to the familiar form of Equation 3.

$$p_D - p_S = \rho_L gh \quad (3)$$

But Equation 2 shows that $p_D - p_S = 1/2 \rho_F v_F^2$. Thus, because the left-hand members of Equations 2 and 3 are identical, the right-hand members of each are also identical. The equating of right-hand members leads to Equation 4.

$$\frac{1}{2} \rho_F v_D^2 = \rho_L gh \quad (4)$$

The significant feature of Equation 4 is that the left-hand member is the kinetic energy loss by the fluid stream in the central tube and the right-hand member represents increase in gravitational potential energy stored in the manometer liquid. When Equation 4 is solved for velocity of fluid (v_D), Equation 5 results.

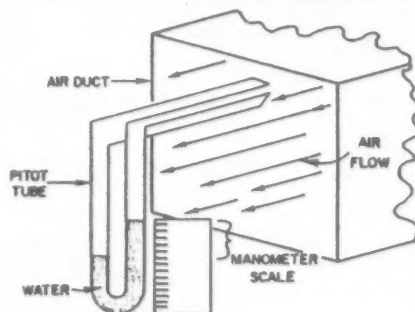
$$v_D = \sqrt{2g \left(\frac{\rho_L}{\rho_F} \right) h} \quad (5)$$

Equation 5 is used in the laboratory experiment in this module and is referred to as the pitot-tube equation. Also, Example A is provided to show how this equation is applied to an air-velocity problem.

EXAMPLE A: AIR VELOCITY MEASUREMENT

Given: A pitot tube is used to measure the velocity of dry air at a point in a large duct. The manometer is filled with water, and the differential height read from the manometer scale is 1 centimeter.

Find: Air velocity in m/s.



Solution:

$$v_D = \sqrt{2 g \left(\frac{\rho_L}{\rho_F} \right) h}$$

Density of water (ρ_L): 1 g/cm³.

Density of air (ρ_F): 0.00129 g/cm³ (dry air).

Therefore,

$$\begin{aligned} v_D &= \sqrt{2 (980 \frac{\text{cm}}{\text{s}^2}) \left(\frac{1 \text{ g/cm}^3}{0.00129 \text{ g/cm}^3} \right) (1 \text{ cm})} \\ &= \left(1233 \frac{\text{cm}}{\text{s}} \right) \left(\frac{10^{-2} \text{ m}}{1 \text{ cm}} \right) \\ v_D &= 12.33 \text{ m/s} \end{aligned}$$

PRACTICAL CONSIDERATIONS FOR MEASUREMENTS

Pitot tubes range from 1/8 inch to 1 inch in diameter. The main disadvantage is that the tube can be used to measure velocities only at one point at a time in the cross section of a pipe or duct, as shown in Figure 3. As a fixed instrument, the pitot tube would not provide an accurate measure of flow through the pipe. If a velocity distribution is desired, however, the pitot tube may be positioned at different radial locations across the pipe; and the data obtained may be used to produce a velocity profile for the fluid flow. In industry, the pitot tube is used primarily as an exploratory device and is rarely implanted permanently in a piping system. It also becomes clogged very easily with foreign matter in the fluid flow. No appreciable pressure loss is caused by insertion of a pitot tube into a fluid stream. It can be inserted into a small hole through the pipe or duct, without requiring much system modification.

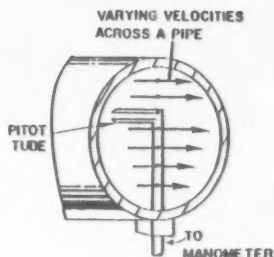


FIG 3

Pressure drops created in the pitot tube are small and require sensitive differential-pressure meters or manometers. Frequently, a slanted manometer is used to increase sensitivity. Water is a light, inexpensive manometer fluid when air velocity measurements are made. Colored alcohol also can be used.

Precautions should be taken to ensure that the axis of the pitot tube always is aligned in the direction of the fluid flow. A component of the fluid stream striking on the static pressure ports will cause pressure inaccuracies to occur; however, the misalignment angle would have to be greater than 10 or 15 degrees to produce significant inaccuracies.

AVIATION APPLICATIONS

The traditional aviation pitot head shown in Figure 4 is a design variation of the basic pitot tube. Dynamic pressure is picked up through port A and is transferred through tube D to the cockpit Airspeed Indicator and to the Mach Number sensing units. Baffle plate B reduces water intake, and water trap C stops any remaining water and exits it through drain holes E and F.

Static pressure is picked up through holes G and H, and is carried through tube I to appropriate instruments. Electrical connection L provides current to electric heaters J and K to prevent ice formation. Many modern aircraft are equipped with pitot heads that pick up only the dynamic pressure. Static pressure is obtained through ports on the fuselage.

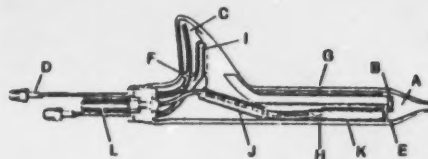


FIG. 4

True airspeed is determined only when these factors are added to the dynamic pressure: (1) the temperature of air in which aircraft is flying and (2) the pressure altitude at which the plane is flying above sea level. The all-important true airspeed is calculated by use of Equation 6.

$$v_t = v_i \frac{T_h + kp_s}{T_s + kp_h} \quad (6)$$

where

- v_t = true airspeed.
- v_i = indicated airspeed.
- T_s = standard temperature (59°F or 15°C).
- p_s = standard pressure (29.92 in. Hg or 760 mm Hg).
- T_h = air temperature at flight altitude.
- p_h = air pressure at flight altitude, in. or mm Hg.
- k = constant value for changing temperatures to absolute values (459.4 for °F and 273 for °C).

SUMMARY

Fluid striking a pitot tube possesses kinetic energy that causes a measurable pressure differential. The tube transfers this pressurized fluid to a measuring device (manometer or other instrument), which, in turn, indicates the velocity either of the fluid flow or of the object (such as an aircraft) passing through the fluid.

LABORATORY

DETERMINATION OF AIR SPEED WITH A PITOT TUBE

MATERIALS

Pitot tube differential gage with manometer and scale (available from Central Scientific Company).
Automobile with speedometer.
Meter stick.

PROCEDURE

1. Convert all velocities in column 1 of Data Table to centimeters per second; record in column 2 of Data Table.
2. Solve Equation 5 for h . Using this equation, calculate predicted manometer difference for each velocity indicated in column 1. Record results of calculations in column 3.
3. An automobile is required. Team with another class member; designate one as driver and other as technician. Each team member has certain duties to perform, as shown in Figure 5. Driver should be concerned with safe driving practices and occasional glances at car speedometer during experiment. Technician is responsible for positioning pitot tube some 2 or 3 ft from car, out of open window on passenger side. Technician must also align pitot tube in car's forward direction and maintain this alignment throughout experiment. Technician must read and record manometer differential heights at various speeds.
4. Auto driver should choose a fairly straight and safe stretch of road and hold a constant speed of 10 miles per hour. When auto is moving at 10 miles per hour, driver signals technician, who reads measured manometer difference (h_m) and records both speedometer reading and h_m in Data Table.
5. Team should repeat step 4 for 30 and 50 miles per hour.
6. If possible, repeat steps 4 and 5 moving in opposite direction; record in Data Table.

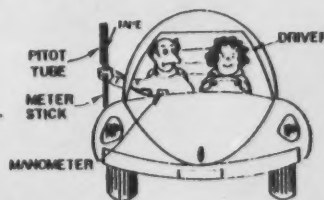


FIG. 5

7. Calculate measured air speed for each speedometer reading, using Equation 5; record in Data Table.
8. Compare measured air speed with speedometer readings.
9. Note weather conditions; record significant conditions causing error in air-speed calculations.
10. Explain the operation of the pitot tube used in this experiment.

DATA TABLE				
Velocity Of Air Stream (mi/h) (Speedometer Reading)	Velocity Of Air Stream (cm/s)	Predicted Manometer Difference (h_p) (cm)	Measured Manometer Difference (h_M) (cm)	Measured Air Speed (mi/h)
10				
30				
50				

SELF CHECK

1. Describe, with the aid of diagrams, the construction and operation of a pitot tube and a manometer suitable for use with the pitot tube.
2. List merits and limitations of the industrial type of pitot tube.
3. A pitot tube used to measure air velocity in a wind tunnel is connected to a manometer that indicates a differential head of 0.20 in of water. Compute air velocity in ft/s if the density of air is 0.075 lb/ft³.
4. To calibrate a pitot tube, compute the theoretically-expected air velocity for each head recorded.

head (h) - inches of water	3.04	6.92	9.63	14.00
velocity (v) - ft/s				

Why would this theoretical velocity not be achieved in actual practical measurements with this pitot tube?

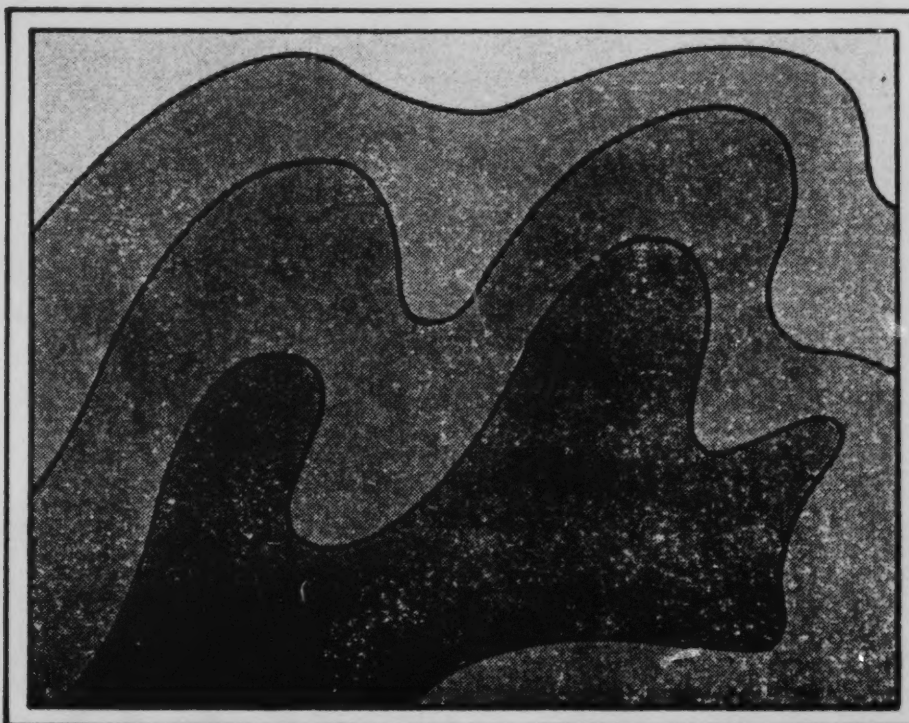
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UNIFIED TECHNICAL CONCEPTS

MODULE 7 F 4
ENERGY CONVERSION
IN FLUID SYSTEMS



FLUID APPLICATION OF
POTENTIAL AND KINETIC ENERGY

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

Energy has been defined as the capacity to do work. Also, energy cannot be created, but energy may be transferred from one form to another within a system.

Potential energy, the capacity to do work because of position, may be transferred to kinetic energy, the capacity to do work because of motion. This module will demonstrate this energy transfer in fluid systems.

PREREQUISITES

Student should have completed Concept Module 7-0, "Potential and Kinetic Energy."

OBJECTIVES

To complete this module, the student should be able to:

- Define:
 - Energy.
 - Mechanical work.
 - Fluid work.
 - Potential energy.
 - Kinetic energy.
- Calculate potential energy of water stored in an overhead reservoir given the weight of water stored and height of the reservoir.
- Calculate fluid work or kinetic energy available for water flowing from an overhead reservoir, given its mass and velocity.
- Calculate mechanical work done by a motor or turbine that pushes water, given net force applied by the motor and distance through which the net force moves.

DISCUSSION

Potential energy is defined as the ability an object possesses, because of its position, to accomplish work. Potential energy may be thought of as energy in storage, waiting to be released.

As an example of stored energy, consider the spring in a clock. Work is done on the spring by winding. This energy is in storage and gradually released in the time operation of the clock. Thus, stored energy is transferred back into a form of work, movement of the internal gears of the clock.

Another example of stored energy is potential energy possessed by water stored in an overhead reservoir. Of course, work must be done to pump water into the overhead reservoir. Once there, however, the water possesses potential energy because of its position. Potential energy of this water may be calculated by the formulas:

$$\text{Potential energy} = \text{weight} \times \text{height.}$$

(1)

$$PE = wh.$$

or

$$\text{Potential energy} = \text{mass} \times \text{acceleration of gravity} \times \text{height.}$$

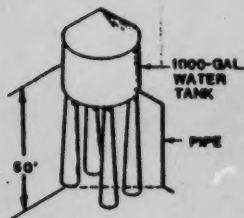
(2)

$$PE = mCh.$$

In the English system of measurement, units of potential energy are foot-pounds, since weight is expressed in pounds and height in feet. In the metric system, potential energy is given in units of newton-meters, since weight (the product of mass times the acceleration of gravity) is expressed in newtons and height in meters. An example of how to calculate the potential energy of a system is shown in Example A.

Kinetic energy may be defined as the ability to do work that an object possesses because of its motion. As an example of kinetic energy, consider water falling over a dam and then used to drive a turbine. The falling water, of course, has both mass and velocity. Kinetic energy may be calculated in terms of these quantities by the formula:

Recall that a gallon of water weighs 8.337 pounds.



EXAMPLE A: CALCULATION OF POTENTIAL ENERGY

Given: 10,000 gallons of water stored in an overhead tank 50 feet above ground level (assume that 50 feet is an "average" height for the 10,000 gallons).

Find: Potential energy of the system.

Solution: (in British units)

$$PE = wh$$

$$= (10,000 \text{ gal})(8.337 \text{ lb/gal})(50 \text{ ft})$$

$$= (83,370 \text{ lb})(50 \text{ ft})$$

$$PE = 4.17 \times 10^6 \text{ ft-lb.}$$

$$\text{Kinetic energy} = \frac{1}{2} \text{ mass} \times \text{velocity}^2.$$

(3)

$$KE = \frac{1}{2} mv^2.$$

In the English system of measurement, units of kinetic energy are foot-pounds when mass is expressed in slugs and velocity in feet per second. In the metric system, units of kinetic energy are newton-meters when mass is expressed in kilograms and velocity in meters per second. A calculation of kinetic energy is given in Example B.

Recall that the weight of 1 slug is 32 pounds and that the dimensional equivalent of a slug is $\frac{1 \text{ lb} \times \text{s}^2}{\text{foot}}$.

EXAMPLE B: CALCULATION OF KINETIC ENERGY

Given: A water balloon of 1 pound of water thrown at a velocity of 20 feet per second.

Find: Kinetic energy of the water balloon.

Solution:

$$KE = \frac{1}{2} mv^2$$

$$= (0.5)(1 \text{ lb})\left(\frac{1 \text{ slug}}{32 \text{ lb}}\right)(20 \frac{\text{ft}}{\text{s}})^2$$

$$= (0.5)\left(\frac{1}{32} \text{ slug}\right)(400 \frac{\text{ft}^2}{\text{s}^2})$$

$$= 6.25 \text{ slug} \frac{\text{ft}^2}{\text{s}^2}$$

$$KE = 6.25 \text{ ft-lb.}$$



Potential energy of water stored in an overhead container can be transferred into the kinetic energy of falling water and this in turn transferred to mechanical work by driving a water motor or water turbine. Example C and the following discussion illustrate this idea.

EXAMPLE C: CALCULATION OF POTENTIAL ENERGY

Given: An overhead reservoir, 20 meters above ground, filled with 500 kilograms of water. (Again note that height of all water in the reservoir is taken to be the average height of 20 m.)

Find: PE of the system.

Solution: (in mks units.) From Equation 2,

$$\begin{aligned} PE &= mgh \\ &= (500 \text{ kg})(9.8 \text{ m/s}^2)(20 \text{ m}) \\ &= 98,000 \text{ kg}^2/\text{s}^2 \\ PE &= 98,000 \text{ newton-meters.} \end{aligned}$$

Recall that the acceleration of gravity is 9.8 m/s^2 and that the dimensional equivalent of a newton is: 1 kg/s^2 .



If the water in the reservoir in Example C is allowed to fall back to ground level, its potential energy is transferred into kinetic energy. Without frictional losses, the transformation is complete, as indicated by Equation 4.

$$mgh = \frac{1}{2} mv^2$$

or

$$v = \sqrt{2gh} \quad (4)$$

where

- v = velocity of object.
- g = acceleration of gravity.
- h = height through which object falls.

Therefore, velocity of the falling water in Example C is:

$$\begin{aligned}
 v &= \sqrt{2 gh} \\
 &= \sqrt{2 (9.8 \text{ m/s}^2) (20 \text{ m})} \\
 &= \sqrt{392 \text{ m}^2/\text{s}^2} \\
 v &= 19.8 \text{ m/s.}
 \end{aligned}$$

Kinetic energy of the falling water just as it reaches ground level is

From Equation 3

$$\begin{aligned}
 KE &= \frac{1}{2} mv^2 \\
 &= (0.5) (500 \text{ kg}) (19.8 \text{ m/s})^2 \\
 &= 98,000 \text{ kg}^2/\text{s}^2 \\
 KE &= 98,000 \text{ newton-meters.}
 \end{aligned}$$

This equals potential energy stored in the water reservoir as calculated in Example C. This demonstrates transformation of potential energy of the water in the reservoir into kinetic energy by the time it falls back to ground level. Use of these equations is further illustrated in Example D.

Recall that the acceleration of gravity is 32 ft/s².

EXAMPLE D: KINETIC ENERGY CALCULATION OF A GRAVITY ATTRACTED MASS

Given: 500 pounds of water falling from a dam spillway 40 feet above a water wheel.

Find: The kinetic energy of the water.

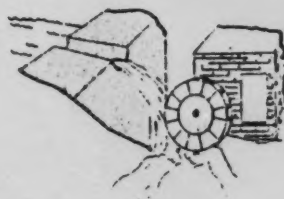
Solution: (from Equation 4)

$$\begin{aligned}
 v &= \sqrt{2 Gh} \\
 &= \sqrt{2 (32 \text{ ft/s}^2) (40 \text{ ft})} \\
 &= \sqrt{2560 \text{ ft}^2/\text{s}^2}
 \end{aligned}$$

$$v = 50.6 \text{ ft/s.}$$

(From Equation 3)

$$\begin{aligned}
 KE &= \frac{1}{2} mv^2 \\
 &= (0.5) (500 \text{ pounds}) \left(\frac{1 \text{ slug}}{32 \text{ pounds}} \right) (50.6 \text{ ft/s})^2 \\
 KE &= 20,000 \text{ ft-lb.}
 \end{aligned}$$



In the confined space of a laboratory, it would be extremely difficult, if not impossible, to store water in an overhead reservoir at sufficient height to develop any appreciable kinetic energy by falling. For this reason, the experiment will use a simulated overhead reservoir. This can easily be done by taking advantage of water pressure available at the lab water faucet.

Pressure developed at the bottom of any liquid storage reservoir can be calculated by the formula:

Pressure = height of water in reservoir x weight density.

$$P = hd. \quad (5)$$

Assuming the kinetic energy of a moving fluid is transferred completely into work, the fluid work formula may be used to calculate kinetic energy of the falling water by the equation:

Kinetic energy = volume flow x pressure.

$$KE = vol \times P. \quad (6)$$

Recall that the mass density of water is 1 g/cm³ or 1000 kg/m³ and the weight of 1 kg is 9.8 newtons.

Can you show that the right side of this equation is another way of writing $PE = mgh$?

It has already been shown in this module that 500 kg of water stored 20 m above ground level has a potential energy of 98,000 newton-meters, which can be transferred to an equal amount of kinetic energy by free falling back to ground level. This same demonstration can be made using the fluid work (or available kinetic energy) formula. Pressure at the bottom of a pipe connected to the overhead reservoir would be:

From Equation 5

$$\begin{aligned} P &= hd \\ &= (20 \text{ m})(9800 \text{ newtons/m}^3) \\ P &= 196,000 \text{ newtons/m}^2. \end{aligned}$$

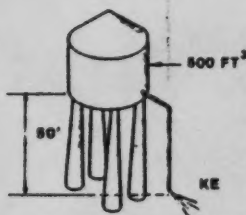
Volume of the 500 kg would be 0.50 m³. Therefore, kinetic energy of the falling water would be:

$$\begin{aligned} KE &= vol \times P \\ &= (0.50 \text{ m}^3)(196,000 \text{ newtons/m}^2) \\ KE &= 98,000 \text{ newton-meters.} \end{aligned}$$

From Equation 6

This is, of course, the same value previously calculated (Example E).

Recall that the weight density of water is 62.4 lb/ft^3 .



EXAMPLE E: CALCULATION OF KINETIC ENERGY OF FLOWING WATER AT BOTTOM OF A RESERVOIR

Given: Water flowing from the bottom of a reservoir 50 feet high and containing 500 cubic feet of water.

Find: Available kinetic energy.

Solution:

$$P = hd$$

$$= (50 \text{ ft})(62.4 \text{ lb/ft}^3)$$

$$P = 3120 \text{ lb/ft}^2$$

$$KE = vol \times P$$

$$= (500 \text{ ft}^3)(3120 \text{ lb/ft}^2)$$

$$KE = 15.6 \times 10^5 \text{ ft-lb}$$

The kinetic energy of water flowing from an overhead reservoir may be converted to mechanical work provided suitable equipment is used. A water motor or water turbine used in conjunction with a prony brake is one such device. A water turbine is simply a paddle wheel fixed to a shaft that is free to rotate. The flowing water turns the paddle wheel and hence also the shaft (Figure 1). Rotational mechanical work may be determined using a prony brake.



Fig. 1



Fig. 2

The prony brake consists of a belt attached to two spring scales and a pulley attached to the shaft of the water motor as shown in Figure 2. As the water motor turns, a frictional force is developed between the belt and the pulley. This force acting through the distance given by the circumference of the pulley is the amount of work done in one revolution. The difference in the spring scale readings F_1 and F_2 gives the net frictional force. Therefore, the total amount of work done in N revolutions can be found from the formula:

$$\text{Work} = \text{net force} \times \text{distance}.$$

$$\text{Work} = (F_1 - F_2) \pi DN. \quad (7)$$

See Example F.

EXAMPLE F: CALCULATION OF PRONY BRAKE WORK

Given: A prony brake that turns through 200 revolutions. The spring scale readings are 18 ounces and 6 ounces, and diameter of the pulley is 3 inches.

Find: Work accomplished.

Solution:

$$\text{Work} = (F_1 - F_2) \pi D N$$

$$= (18 \text{ oz} - 6 \text{ oz}) (3.14) (3 \text{ in}) (200)$$

$$= (12 \text{ oz}) (1884 \text{ in})$$

$$= 22,600 \text{ in-oz}$$

$$= (22,600 \text{ in-oz}) \left(\frac{1 \text{ ft}}{12 \text{ in}} \right) \left(\frac{1 \text{ lb}}{16 \text{ oz}} \right)$$

$$\text{Work} = 118 \text{ ft-lb.}$$

LABORATORY

MATERIALS

Water motor equipped with prony brake, pressure gage and revolution counter (such as Cenco No. 76250-000 or similar supplier).
Vernier caliper.
Catch bucket (minimum 5-gallon capacity).
Bathroom scales.

PROCEDURE

1. Set up equipment as shown in Figure 3.
2. Use a vernier caliper to measure diameter of pulley on prony brake; record in data table.
3. Measure catch bucket weight; record in data table. Also measure and record bucket diameter. Set bucket to one side ready for use.
4. Loosen prony brake clamp; adjust belt tension so that both scales on prony brake read zero.
5. Open water faucet; adjust flow so that water motor just begins to turn. Observe pressure gage reading; record in data table.
6. Adjust belt tension carefully so that a noticeable drag is applied to prony brake but not enough to stall water motor. Tighten clamp.
7. Observe reading on both scales; record in data table.
8. Observe initial reading on revolution counter; record in data table.
9. Slip catch bucket under water motor outlet; at the same time, engage revolution counter.
10. When catch bucket nears its capacity, carefully slip it from under water motor outlet; at the same time, disengage revolution counter.
11. Observe final reading on revolution counter; record in data table.
12. Measure weight of full catch bucket; record in data table. Also measure and record depth of water in bucket. Empty

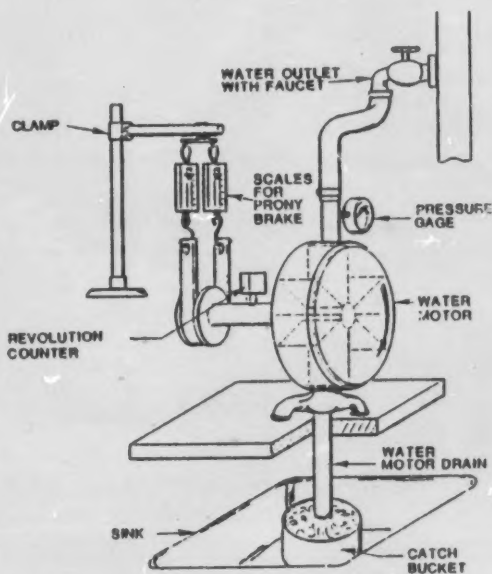


Fig. 3

- bucket; set it aside ready for use again.
13. Calculate mechanical work output by water motor, kinetic energy input to water motor and simulated potential energy available from an overhead storage tank. Complete data table for first trial.
 14. Repeat steps 4 through 13 for remaining trials at gradually increasing water pressures.

Recall that pressure = height x weight density. Therefore, height = pressure ÷ weight density.

DATA TABLE										
Diameter of prony brake pulley _____ inches = _____ ft.										
Weight of catch bucket _____ lb.										
Diameter of catch bucket _____ inches = _____ ft.										
Trial	(1) Pressure Gage Reading (lb/in ²)	(2) Scale Reading F ₁ (lb)	(3) Scale Reading F ₂ (lb)	(4) Rev Counter At Start	(5) Rev Counter At End	(6) Wt Full Catch Bucket (lb)	(7) Depth of Water in Bucket (ft)	Mech Work Output (ft-lb)	Kinetic Energy Available (ft-lb)	Simulat Potenti Energy (ft-lb)
1										
2										
3										
4										
5										
6										

$$\text{Mechanical work} = (F_1 - F_2) \pi DN$$

$$\text{Kinetic energy available} = \text{vol} \times P = \left[\frac{\pi}{4} (\text{diameter of bucket})^2 (\text{column 7}) \right] (\text{column 1}) (144 \text{ in}^2/\text{ft}^2).$$

$$\text{Simulated potential energy} = wh = (\text{column 6} - \text{wt of catch bucket}) \left[\frac{(\text{column 1}) (144 \text{ in}^2/\text{ft}^2)}{62.4 \text{ lb/ft}^3} \right].$$

SELF CHECK

1. Define:
 - Energy.
 - Potential energy.
 - Kinetic energy.
 - Mechanical work.
 - Fluid work.
2. What is the potential energy of 50,000 gallons of water stored in an overhead reservoir at an average height of 50 feet above ground level?
3. How much kinetic energy is available for conversion to work when 1000 gallons of water at a pressure of 80 lb/in² flows through a turbine?
4. What pressure is developed in a connecting pipe 30 feet below an overhead storage tank filled with water?

REFERENCES

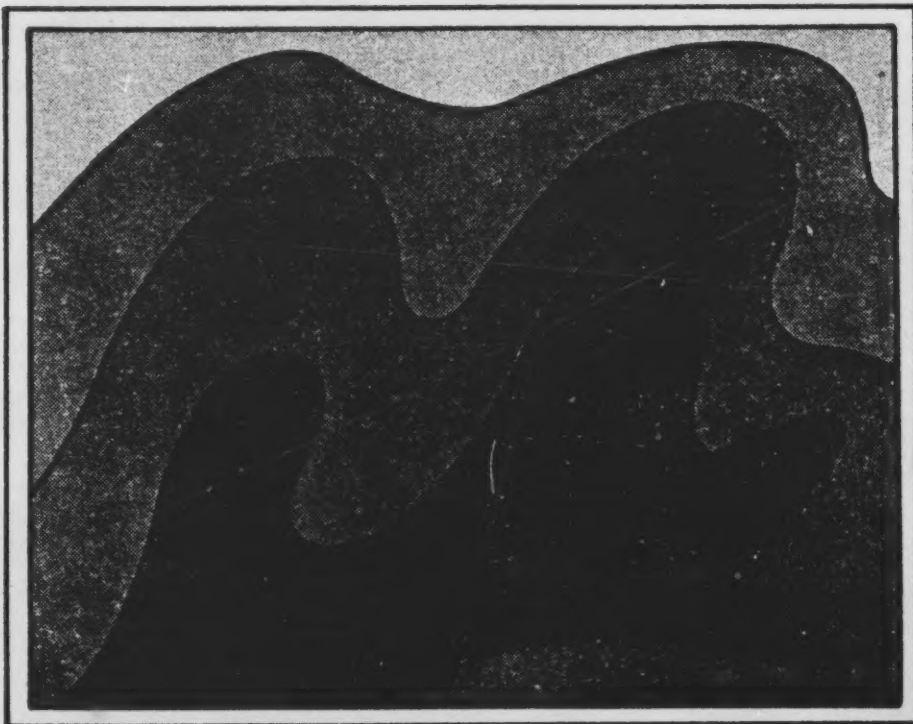
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UNIFIED TECHNICAL CONCEPTS

MODULE 7 F 5

GAS LAWS



FLUID APPLICATION OF POTENTIAL AND KINETIC ENERGY

Technical Education Research Center - Southwest

TEC - SW

INTRODUCTION

GAS LAWS

Gases possess kinetic energy in the form of moving molecules that collide with one another and with objects in which they are contained, causing pressure that increases as the temperature increases. Many devices, such as the internal combustion engine, involve expansion or compression of gases. This module presents instructions that illustrate the solution of many gas problems, although chemical reaction and departure from ideal gas behavior are not included. In the laboratory, the student will measure, calculate, and plot gas pressure and temperature.

PREREQUISITES

Student should have completed Concept Module 7-0, "Potential and Kinetic Energy," and Application Modules 1F1, "Atmospheric Pressure," and 1F4, "Manometers."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Explain the relationship between kinetic energy and molecular motion in a gas.
- Define the General Gas Law.
- Describe these three gas laws:
 - Boyle's Law.
 - Charles' Law.
 - Gay-Lussac's Law.
- Define the concept of an ideal gas.

- Given the initial pressure and temperature of a given volume of air, calculate the pressure when the temperature increases to a given point.
- Given the appropriate equipment, measure, and plot gas pressure versus volume at a fixed temperature.
- Given the appropriate equipment, measure and plot gas pressure versus temperature for a fixed volume of gas.

DISCUSSION

All gases possess kinetic energy because of the motion of their molecules. Molecules of a solid vibrate and rotate about fixed positions; those of a liquid slide over and collide with one another and with the walls of the containers holding them. Gas molecules, however, are farther apart and move at greater speeds than those in a solid or a liquid. A gas is, therefore, greatly compressible, in spite of the constant molecular motion.

Such molecular motion within a gas is illustrated by Figure 1, where molecules are shown striking the container and its movable, air-tight top with such force that a weight upon the top is suspended, or balanced, there. The combined force of the top and the weight are compressing the gas until balanced by the upward force exerted by the gas.

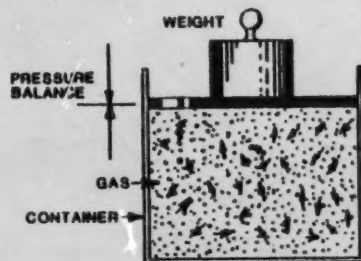


FIG. 1

Pressure is a force per unit area, and the force to contain gas is the force necessary to "reflect" gas atoms or molecules from the walls of the container and to reverse their momentum. Momentum is the product of the mass and velocity of the atom ($p = mv$), and velocity is a function of the temperature of the gas. A higher temperature causes a higher molecular velocity. The temperature of a gas is proportional to the average kinetic energy ($\frac{1}{2}mv^2$) of its molecules; thus, T is proportional to v^2 . Consequently, when the temperature of a gas is raised, the pressure increases if the volume does not change, as shown by the tire in Figure 2. Two views of the same tire are shown: In Figure 2a the tire is cold and almost flat, while in Figure 2b the same tire, containing the same air, is fully inflated. An increase in temperature caused increased molecular speed and air pressure. An automobile actually rides upon the pressure created by millions of molecules within the tires (containers).

THE GENERAL GAS LAW

The General Gas Law relates absolute pressure, volumes, and absolute temperatures of a fixed mass of gas, as stated in Equation 1.

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} = \frac{p_3 V_3}{T_3} = C \quad (1)$$



FIG. 2

where

p_1 = pressure, initial.

V_1 = volume, initial.

T_1 = absolute temperature, initial.

C = a constant.

subscripts 2 and 3 = later states.

For any condition, therefore, Equation 1 can be written as Equation 2.

$$\frac{pV}{T} = C \quad (2)$$

Equation 2 assumes that there is no change in the amount of gas and no change of state (such as condensation).

THREE SPECIAL FORMS OF THE GENERAL LAW

Boyle's Law, Charles' Law, and Gay-Lussac's Law are special applications of the general gas law.

BOYLE'S LAW

Boyle's Law is designed for a constant temperature process; it is indicated by Equation 3.

$$pV = C \quad (3)$$

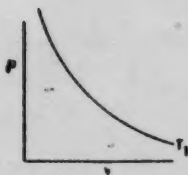


FIG. 3

As displayed in Figure 3, decreases in volume are accompanied by an increase in pressure, or a decrease in pressure results in an increase in volume, when the temperature is held constant. The relationship between pressure and volume first was stated as a law in 1662 by Robert Boyle, after whom it was named.

Equation 3 is utilized in solving the problem in Example A.

EXAMPLE A: SURGE TANK PRESSURE AND BOYLE'S LAW

Given:

An 85-gallon surge tank is installed in a water system, as illustrated in the drawing below. The tank is 63" high and 20" in diameter. Pressure switches in the surge tank start the pump at a pressure of 20-psig (gage) and stop the pump when the pressure reaches

several hours and air temperature in the tire has increased to 170°F.

Solution: $p_1 = 25 \text{ psig} + 14.7 \text{ psia} = 39.7 \text{ psia}$
 $T_1 = 70^\circ\text{F} + 460 = 530^\circ\text{R}$ (converting to an absolute
 $T_2 = 170^\circ\text{F} + 460 = 630^\circ\text{R}$ temperature scale)

Assume that the volume of the tire remains essentially constant, Equation 5 is used as follows:

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

or

$$p_2 = \frac{p_1 T_2}{T_1}$$
$$= \frac{(39.7 \text{ psia})(630^\circ\text{R})}{(530^\circ\text{R})}$$
$$p_2 = 47.2 \text{ psia}$$

or

$$p_2 = 47.2 \text{ psia} - 14.7 \text{ psia}$$
$$p_2 = 32.5 \text{ psig}$$

THE IDEAL LAW

The concept of an ideal, imaginary gas is useful because it permits its behavior to be predicted. Behavior of an actual gas is predictable from ideal gas laws also, to the extent that the gas conforms to the requirements of the ideal gas. At temperatures much higher than the boiling temperature, and at normal or low pressures, most real gases come very close to satisfying the conditions of an ideal gas. For most gases, these conditions are satisfied at 70°F and at normal atmospheric pressure. However, steam must be "superheated" to several hundred degrees before its behavior approximates that of an ideal gas.

Thomas Andrews investigated the pressure-volume relationships of CO₂ at Queen's College, Belfast, between 1861 and 1869. He observed, as indicated in Figure 6, that as he lowered the temperature, the smooth curves began developing irregularities that eventually led to a change in volume without a change in pressure. Andrews accurately concluded that the gas actually was changing to a liquid under these conditions. The highest temperature at which the gas can liquefy is the "critical temperature"; the pressure at the critical point (Figure 6) is the "critical pressure."

The "critical point" indicates where to expect the failure

And,

$$V_H = 19792 - 6178$$

$$V_H = 13614 \text{ in}^3$$

Then,

$$h_2 = \frac{4 V_H}{\pi D^2}$$

$$h_2 = \frac{4 (13614 \text{ in}^3)}{\pi (20 \text{ in})^2}$$

$$h_2 = 43.3 \text{ in}$$

CHARLES' LAW

Charles' Law is designed for a constant pressure process, and is stated in Equation 4.

$$\frac{V}{T} = C \quad (4)$$

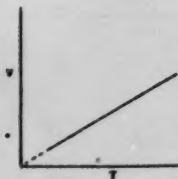


FIG. 4

Increases in temperature cause an increase in volume when the pressure is maintained at the same value. This relationship is named after its discoverer, Jacques Charles. Figure 4 indicates how pressure is plotted against volume and temperature using Charles' Law.

GAY-LUSSAC'S LAW

Gay-Lussac's Law is for a constant-volume process, as plotted in Figure 5 and as stated in mathematical terms by Equation 5.



FIG. 5

$$\frac{P}{T} = C \quad (5)$$

Increases in temperature of a gas when the volume is held constant cause an increase in pressure.

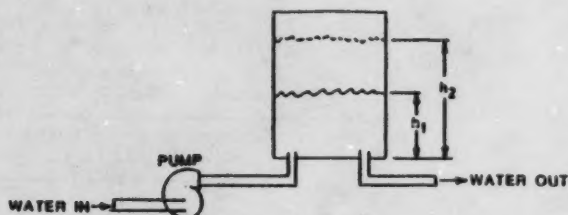
Equation 5 is used to solve a problem in Example B.

EXAMPLE B: TIRE PRESSURE AND GAY-LUSSAC'S LAW

Given: An automobile tire with a volume of 1500 in³ is pressurized to 25 psig at a temperature of 70°F.

Find: Pressure in the tire after the car has been driven

40-psig. The space above the water (called "ullage") is filled with air. The temperature of the air remains essentially constant during this process. The water is 32" high when the pressure is 20 psig.



Find: Water height when the pressure is 40 psig.

Solution: Symbols used in calculations:

psia = pounds per sq in absolute.

V_T = total tank volume.

V_L = volume of water at h_1 .

V_H = volume of water at h_2 .

V_1 = volume of air when water volume is V_L .

V_2 = volume of air when water volume is V_H .

$$V_T = \frac{\pi}{4} d^2 h_T = \frac{\pi}{4} (20 \text{ in})^2 (63 \text{ in})$$

$$V_T = 19792 \text{ in}^3$$

$$V_L = \frac{\pi}{4} d^2 h_1$$

$$V_L = \frac{\pi}{4} (20 \text{ in})^2 (32 \text{ in})$$

$$V_L = 10053 \text{ in}^3$$

$$V_1 = V_T - V_L$$

$$V_1 = 19792 - 10053 = 9739 \text{ in}^3$$

$$p_1 = 20 \text{ psig} + 14.7 \text{ psia} = 34.7 \text{ psia}$$

$$p_2 = 40 \text{ psig} + 14.7 \text{ psia} = 54.7 \text{ psia}$$

From Equation 3:

$$p_1 V_1 = p_2 V_2$$

or

$$\frac{p_1}{p_2} = \frac{V_2}{V_1}$$

Therefore,

$$V_2 = V_1 \frac{p_1}{p_2}$$

So,

$$V_2 = (9739 \text{ in}^3) \left(\frac{34.7 \text{ psia}}{54.7 \text{ psia}} \right)$$

$$V_2 = 6178 \text{ in}^3$$

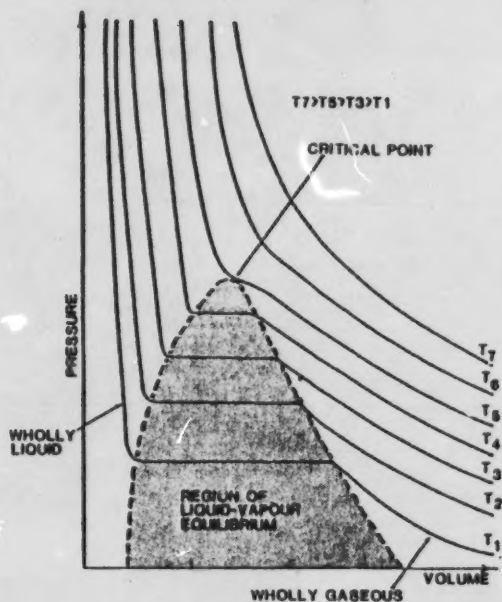


FIG. 6

of the ideal gas law. There is considerable deviation from the ideal gas law for temperatures and pressures near the critical point. Several more sophisticated equations have been developed to represent the behavior of a gas that does not obey the ideal gas law. These equations are empirical and only can approximate the behavior of gases under conditions far removed from their liquefaction points. Such equations are beyond the scope of this module.

Caution must be exercised in the application of the laws expressed in Equations 1 through 5 to assure that the required conditions are not violated. For example, since compression is so rapid in an internal combustion engine, there is no opportunity for appreciable heat transfer. As a result, the temperature of gas in the cylinder is not constant. Thermal processes of this type are called "adiabatic processes." Equation 3, designed for use with constant temperature, is not applicable. In such a case, gas temperature is unknown and is not computed easily from first principles. This problem can be handled by use of Equation 6, which allows for such rapid changes.

$$PV^k = C \quad (6)$$

where
 $k = 1.4$ for air.

Equation 6 and Equation 3 (Boyle's Law) are compared and used to solve a problem in Example C.

EXAMPLE C: INTERNAL COMBUSTION ENGINE PRESSURE

Given: An eight-cylinder, internal-combustion engine has a compression ratio of 10 and a displacement of 350 in³. Assume that the cylinder is filled with air at a pressure of 0 psig (atmospheric pressure) and that no ignition occurs.

Find: Cylinder pressure when the piston is at top dead center, (a) assuming a constant temperature process (Equation 3) and (b) assuming no heat loss (Equation 6).

Solution: Since the engine has a compression ratio of 10, then $\frac{V_2}{V_1} = \frac{1}{10}$; that is, the gas volume after compression is $\frac{1}{10}$ the initial gas volume, assuming that no ignition has occurred.

(a) From Equation 3,

$$p_1 V_1 = p_2 V_2$$

or,

$$\frac{p_1}{p_2} = \frac{V_2}{V_1}$$

Therefore,

$$\frac{p_1}{p_2} = \frac{1}{10}$$

And,

$$p_2 = 10 p_1$$

Since $p_1 = 0 \text{ psig} + 14.7 \text{ psia} = 14.7 \text{ psia}$, then

$$p_2 = 10 (14.7 \text{ psia}) = 147 \text{ psia}$$

or

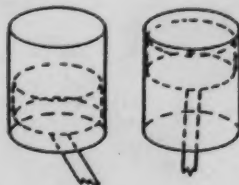
$$p_2 = 147 \text{ psia} - 14.7 \text{ psia} = 132.3 \text{ psig}$$

(b) From Equation 6,

$$p_1 V_1^k = p_2 V_2^k$$

or

$$\frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^k$$



DEMONSTRATION OF GAY-LUSSAC'S LAW - P-T

MATERIALS

Bunsen burner or hot plate.
Insulated metal container having two holes in top.
Thermometer.
Two corks to fit holes in container.
Manometer.
Stand to hold container above burner.
Small-diameter flexible tubing and rigid tubing.

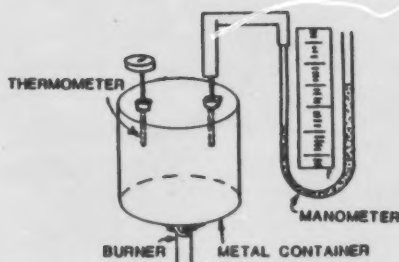


FIG. 8

PROCEDURE

1. Push thermometer through a cork, and insert cork into metal container. (Refer to Figure 8.)
2. Insert a piece of small-diameter, rigid tubing into a cork; install cork in metal container.
3. Place metal container on a stand over Bunsen burner.
4. Connect manometer to tubing in cork in metal container, using flexible tubing.
5. Light burner, and heat air inside container.
6. Measure pressure and temperature of gas, twice as it increases in temperature and twice as it cools. Record in Data Table 2.
7. Plot measured temperature as a function of pressure. Use graph in Data Table 2.
8. Why must the volume of gas in the container remain unchanged during these measurements?
9. Does the graph plotted in Data Table 2 agree with that predicted by the equation $\frac{P}{T} = C$?

LABORATORY

DEMONSTRATION OF BOYLE'S LAW -PV=C

MATERIALS

Bicycle pump.
Manometer.

PROCEDURE

1. Disassemble bicycle pump, and measure inside diameter and stroke.
2. Compute and plot volume of the cylinder as a function of the displacement of pump handle.
3. Reassemble pump.
4. Connect manometer to bicycle pump, in Figure 7.
5. Slowly push in on handle of pump, stopping at four measured distances. Measure pressure at each step. Record distances and pressure at each step in Data Table 1. CAUTION: Do not pull back on pump handle while manometer is connected.
6. Plot measured pressure points as a function of volume. Use graph in Data Table 1.
7. Why must pressure at each distance in Step 5 be measured at the same temperature?
8. Does the graph plotted in Data Table 1 agree with that effected from the equation $pV = C$?

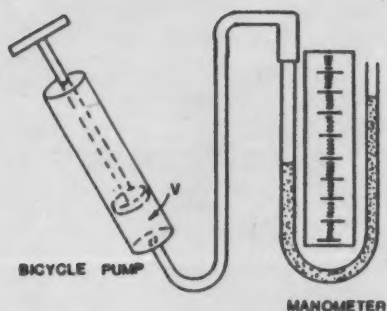


FIG. 7

Therefore,

$$\frac{p_1}{p_2} = \left(\frac{1}{10}\right)^{1.4}$$

$$\frac{p_1}{p_2} = 0.0398$$

Then,

$$p_2 = \frac{p_1}{0.0398} = \frac{14.7 \text{ psia}}{0.0398}$$

$$p_2 = 369.3 \text{ psia}$$

or

$$p_2 = 369.3 \text{ psia} - 14.7 \text{ psia}$$

$$p_2 = 354.6 \text{ psia}$$

A comparison of the two calculated pressures (132.3 psig and 354.6 psig) reveals a considerable difference, which could be an important factor, depending upon how these results are used. If the initial temperature (T_1) and the temperature following compression (T_2) had been measured, then Equation 1 could have been used.

DATA TABLE 1				
	Cylinder Distance (Measured)	Volume (Calculated)	Pressure (Measured)	
#1	_____ in	_____ in ³	_____ psia	
#2	_____ in	_____ in ³	_____ psia	
#3	_____ in	_____ in ³	_____ psia	
#4	_____ in	_____ in ³	_____ psia	

Plot p versus V for four points given above.

p

V

DATA TABLE 2		
	Pressure	Temperature
(Increasing gas temperature)		
1st reading →	_____ psia	_____ °R
2nd reading →	_____ psia	_____ °R
(Decreasing gas temperature)		
3rd reading →	_____ psia	_____ °R
4th reading →	_____ psia	_____ °R

Plot four pair of values on plot shown. Use appropriate scales.



SELF CHECK

1. Explain the relationship between kinetic energy and molecular motion in a gas.
2. State the three properties of a gas related by the General Gas Law.
3. Describe the following gas laws — use words and equations. State what "variable" is held constant in each law.

Boyle's Law.

Charles' Law.

Gay-Lussac's Law.

4. Why is the concept of an "ideal gas" important in studying the behavior of gases?
5. The tires on an automobile are pressurized to 28 psig at a temperature of -30°F . The car is driven across the country to a warmer climate, where the temperature is 110°F . Compute the air pressure in the tires, assuming that the volume of the tires remains essentially constant at 2000 in^3 .

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UNIFIED TECHNICAL CONCEPTS

MODULE 7 F 8

SIPHONS



FLUID APPLICATION OF POTENTIAL AND KINETIC ENERGY

Technical Education Research Center - Southwest

TEC - SW

INTRODUCTION

A siphon is a bent tube or pipe used to carry liquid up and over the edge of a container and then on to a lower elevation, without the use of a pump. Waterlines often act as siphons by transferring water from water tanks over hills to homes at lower elevations. Siphons also are useful in separating liquids of different densities.

In this module, the physical principles that produce flow in a siphon will be explained. The relationship between atmospheric pressure, liquid pressure, elevation, liquid velocity, and fluid resistance in a siphon will be illustrated by the application of Bernoulli's equation to a siphon. In the laboratory, the student will conduct experiments that illustrate the relationship between fluid elevation, flow in the siphon, and fluid resistance in the siphon.

PREREQUISITES

Student should have completed Concept Module 7-0, "Potential and Kinetic Energy," and Application Module IF1, "Atmospheric Pressure."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Draw and label a siphon and explain in three or four sentences how it works.
- Given the height differential of a siphon, calculate the differential pressure.
- Given a siphon that transfers water from an upper container to a lower container so placed that the lower end of the siphon remains above the water level in the lower container, write the Bernoulli equation for flow in the siphon, and calculate the velocity of flow from the siphon.

- State what happens to the velocity of flow as the water level drops in the upper container and to the velocity of flow when the distance is increased between the water level in the upper container and the lower end of the siphon.
- Explain, in three or four sentences, the relationship between flow resistance in the tube and rate of flow through the siphon.
- Given the height differential for a siphon, the diameter of the siphon, the weight density of the fluid, and the frictional loss, calculate the volume flow rate.
- Given the appropriate equipment, construct a siphon; and measure the flow rate of the siphon.

DISCUSSION

In Module 1F1, "Atmospheric Pressure," the pressure at the bottom of a column of liquid was given by Equation 1.

$$p = \rho gh \quad (1)$$

where

p = pressure.

ρ = mass density of liquid.

g = acceleration of gravity.

h = height of liquid column.

When using English units, the weight density (rather than the mass density) of the liquid is used in calculations. Weight density is the product of mass density and the acceleration of gravity, or ρg in Equation 1. Example A illustrates the use of Equation 1 in solving a problem.

EXAMPLE A: HEIGHT OF WATER COLUMN SUPPORTED BY ATMOSPHERIC PRESSURE

Given: Atmospheric pressure is 14.7 lb/in^2 at sea level. The weight density of water is $\rho g = 62.4 \text{ lb/ft}^3$.

Find: The maximum height of a water column that can be supported by atmospheric pressure.

Solution: $p = \rho gh$

$$h = \frac{p}{\rho g}$$

$$= \frac{(14.7 \frac{\text{lb}}{\text{in}^2})(144 \frac{\text{in}^2}{\text{ft}^2})}{(62.4 \text{ lb/ft}^3)}$$

$$h = 33.9 \text{ ft}$$

DIFFERENTIAL PRESSURE IN A SIPHON

Figure 1 shows a siphon used to move water up over the edge of container 1 to container 2 at a lower level. The siphon is filled with water and is clamped at the lower end before it is placed in container 1. Calculation of the differential pressure in the siphon between points A and C is based upon the static pressures that exist before the clamp is removed.

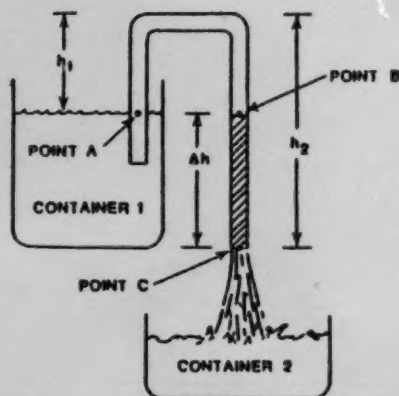


FIG. 1

The pressure at Point A (level with liquid surface) is atmospheric pressure. At points in the tube above Point A, the pressure is less than atmospheric. Since Point B is level with Point A, the pressures at A and B are equal under static conditions. The pressure at Point C is greater than the pressure at Points A and B by an amount equal to $\rho g \Delta h$, where Δh is equal to $(h_2 - h_1)$. The pressure difference clearly is due to the column of liquid between Point A in Container 1 and Point C in Container 2. Thus, the differential pressure in a siphon depends only upon the height of the original liquid level above the discharge point, just as it does in the case of a discharge hole in the side of the container. If the lower end of the siphon extends below the level of the water in Container 2, Point C still is defined to be on the same level as the water level in Container 2. The length of siphon tube extending below the liquid level in either container has no effect upon differential pressure in the siphon.

When the clamp is removed, liquid flow is produced by the force of gravity or weight of water in the siphon. The pressure upon the liquid at point A in the left side of the siphon is equal to that upon the liquid at point B in the right side. The gravitational force upon the liquid below point B on the right side (shaded area) pulls this liquid out of the siphon, therefore momentarily lowering the pressure at point B. Atmospheric pressure upon the liquid at point A in container 1 now momentarily exceeds the pressure at point B and, consequently, forces more liquid into the tube to replace that which has flowed from the tube. As indicated in Example A, this process will fill the tube only if h_1 is less than 34 feet for water at standard atmospheric pressure.

FLOW RATE IN A SIPHON

In Concept Module 7-0, "Potential and Kinetic Energy," Bernoulli's equation was given as follows:

$$p + \frac{1}{2} \rho v^2 + \rho gh = \text{constant} \quad (2)$$

where

p = pressure.
 v = fluid velocity.

133

ρ = mass density of liquid.
 g = acceleration of gravity.
 h = height of liquid above point being considered.

The quantity on the left of the equal sign in Bernoulli's equation is a constant for all points in a fluidal system if friction is ignored. Application of this equation to Points 1 and 2 of Figure 2 yields Equation 3.

$$p_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2 \quad (3)$$

At Point 1, the fluid velocity is nearly zero and can be ignored, and the pressure is atmospheric pressure p_a . At point 2, the pressure also is atmospheric pressure; thus, Equation 3 can be written in the form of Equation 4.

$$p_a + \rho g h_1 = p_a + \frac{1}{2} \rho v_2^2 + \rho g h_2 \quad (4)$$

or

$$\begin{aligned}
 \frac{1}{2} \rho v_2^2 &= \rho g (h_1 - h_2) \\
 v_2^2 &= 2 g (h_1 - h_2) \\
 v_2 &= \sqrt{2 g \Delta h} \quad (5)
 \end{aligned}$$

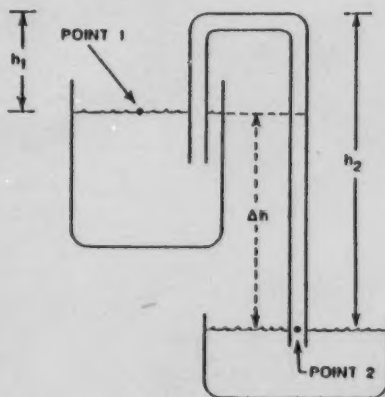


FIG. 2

This equation gives the velocity of fluid flowing from the open end of a siphon if there is no friction. This is the same equation developed in Concept Module 7-0 for fluid velocity through a hole in the side of a container when the hole is a distance Δh below the liquid level. Example B illustrates the use of Equation 5 in solving a problem.

EXAMPLE B: VOLUME FLOW RATE OF A FRICTIONLESS SIPHON

Given: A siphon having a diameter of 2 inches is used to move water through a vertical distance of 20 feet. (Assume no friction.)

Find: Volume flow rate.

Solution: Fluid velocity:

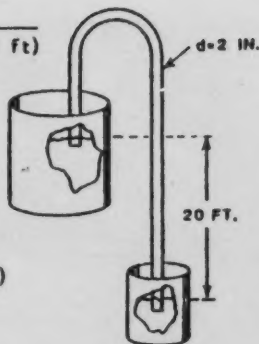
$$\begin{aligned}v &= \sqrt{2gh} \\&= \sqrt{(2)(32 \text{ ft/s}^2)(20 \text{ ft})} \\&= \sqrt{1280 \text{ ft}^2/\text{s}} \\v &= 35.8 \text{ ft/s}\end{aligned}$$

Area of pipe:

$$\begin{aligned}A &= \pi r^2 \\&= (3.14)(1 \text{ in})^2 \\&= (3.14 \text{ in}^2) \left(\frac{1 \text{ ft}^2}{144 \text{ in}^2} \right) \\A &= 0.0218 \text{ ft}^2\end{aligned}$$

Volume flow rate:

$$\begin{aligned}Q_v &= vA \\&= (35.8 \text{ ft/s})(0.0218 \text{ ft}^2) \\Q_v &= 0.78 \text{ ft}^3/\text{s}\end{aligned}$$



If the fluid resistance in a siphon is taken into consideration, Equation 4 can be written in the following form:

$$\rho gh_1 = \frac{1}{2} \rho v_1^2 + \rho gh_2 + F$$

where

F - frictional loss in lb/in^2 .

Note that the frictional loss F is expressed in units of pressure, indicating that the existence of friction in the siphon effectively shows up as a decrease in pressure head driving the fluid, as indicated by Equation 6.

$$F = \rho g \Delta h - \frac{1}{2} \rho v_1^2 \quad (6)$$

Example C illustrates the use of Equation 6 in determining the effect of frictional loss in a siphon.

EXAMPLE C: FRICTIONAL LOSS IN A SIPHON

Given: A siphon is used to transport water through a vertical distance of 4 feet. The velocity of the water at the outlet end of the siphon is 12 ft/s.

Find: Frictional loss in the siphon.

Solution: Weight density of water:

$$\rho g = 62.4 \text{ lb/ft}^3$$

Mass density of water:

$$\rho = \frac{62.4 \text{ lb/ft}^3}{32 \text{ ft/s}^2}$$

$$\rho = 1.95 \text{ slugs/ft}^3$$

Frictional loss:

$$\begin{aligned} F &= \rho g \Delta h - \frac{1}{2} \rho v^2 \\ &= (62.4 \text{ lb/ft}^3)(4 \text{ ft}) - \frac{1}{2} (1.95 \text{ slugs/ft}^3)(12 \text{ ft/s})^2 \\ &= 249.6 - 140.4 \\ &= 109.2 \text{ lb/ft}^2 \end{aligned}$$

$$\begin{aligned} F &= (109.2 \text{ lb/ft}^2) \left(\frac{1 \text{ ft}^2}{144 \text{ in}^2} \right) \\ \therefore F &= 0.76 \text{ lb/in}^2 \end{aligned}$$

A pressure difference of 0.76 lb/in² is required to overcome frictional losses.

SUMMARY

Siphons are used to carry liquids up over the side of a container and down to a lower level without the use of a pump. The maximum allowable height of the siphon tube above the original liquid level is the height of a column of that liquid that can be supported by the pressure applied to its surface. The flow rate supported by the pressure applied to its surface. The flow rate through a siphon is proportional to the square root of the difference in height between the original liquid level and the discharge level of the siphon. Frictional losses reduce flow rate.

LABORATORY

FLOW RATE OF A SIPHON

MATERIALS

Two graduated beakers, 2-liter capacity or greater.
Two pieces of plastic tubing of equal length (approximately 3 ft) and different diameters.
Meter stick.
Lab jack.
Support stand.
Clamp for tubing.
Stopwatch or timer.
Vernier caliper.

PROCEDURE

1. Measure inner diameter of tubing with caliper, and record in Data Table.
2. Set up equipment as illustrated in Figure 3. The support stand holds the outlet end of siphon tube at a fixed level. Fill siphon tube with water, and clamp lower end. Insert upper end in beaker.
3. Release clamp, and measure time required for a given volume of water to flow into lower beaker. Continuously adjust height of upper beaker with lab jack to maintain a constant height Δh of liquid during timed flow. Record time required, volume transferred, and height difference Δh in Data Table (1 liter = 0.0353 ft³). Repeat with two other values for height Δh .
4. Repeat with tubing of a different diameter. Record results.
5. Calculate tubing area in ft² and volume flow rate in ft³/s.
6. Calculate theoretical fluid velocity in ft/s from Equation 5.
7. Calculate actual fluid velocity in ft/s by dividing volume flow rate by tubing area.

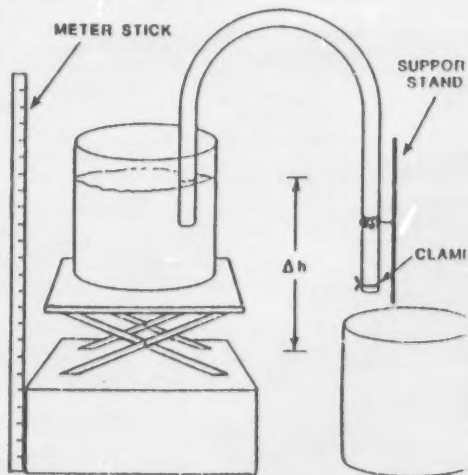


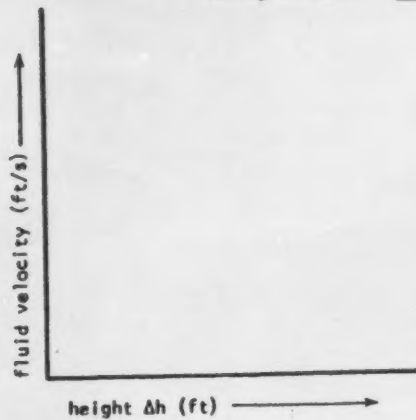
FIG. 3

8. Calculate frictional loss from Equation 6. Mass density of water: $\rho = 1.95 \text{ slugs/ft}^3$.
9. Select appropriate scales, and plot graphs that show theoretical and actual fluid velocity for each diameter tubing as functions of height differential.
10. Discuss flow rate through a siphon by referring to the graphs.

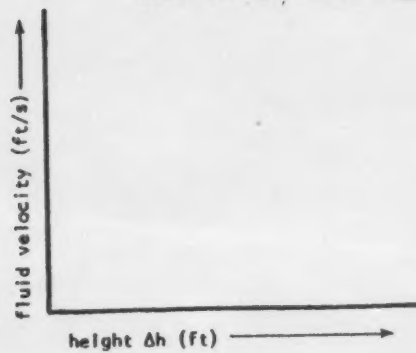
DATA TABLE

Tubing diameter (inner)		Inner Tubing area	Liquid Volume		Time	Height Δh		Volume flow rate	Liquid Velocity in ft/s		Frictional loss
in	ft	ft ²	liter	ft ³	s	in	ft	ft ³ /s	Theoretical	actual	lb/in ²

tubing diameter = _____ in



inner tubing diameter = _____ in

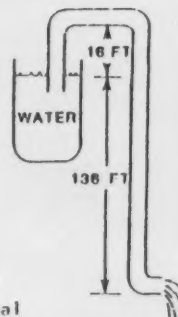


inner tubing diameter = _____ in

140

SELF CHECK

1. Draw and label a siphon, and describe how it works.
2. Determine the differential pressure in the siphon illustrated in the adjoining sketch.
3. Determine the theoretical velocity of the fluid leaving the siphon in problem 2 if friction is ignored.
4. State what happens to the flow rate in a siphon if the water level in the upper container drops. Hint: Refer to Equation 5.
5. Explain the relationships between diameter of a siphon tube, fluid resistance in the tube, and volume flow rate.
6. A siphon 6 inches in diameter is used to transport oil having a weight density of 42.4 lb/ft^3 through a height difference of 25 ft. The volume flow rate is $5 \text{ ft}^3/\text{s}$. Find the frictional loss.
7. The frictional loss in problem 6 is reduced by 25% when the oil is heated. Calculate the resulting volume flow rate.



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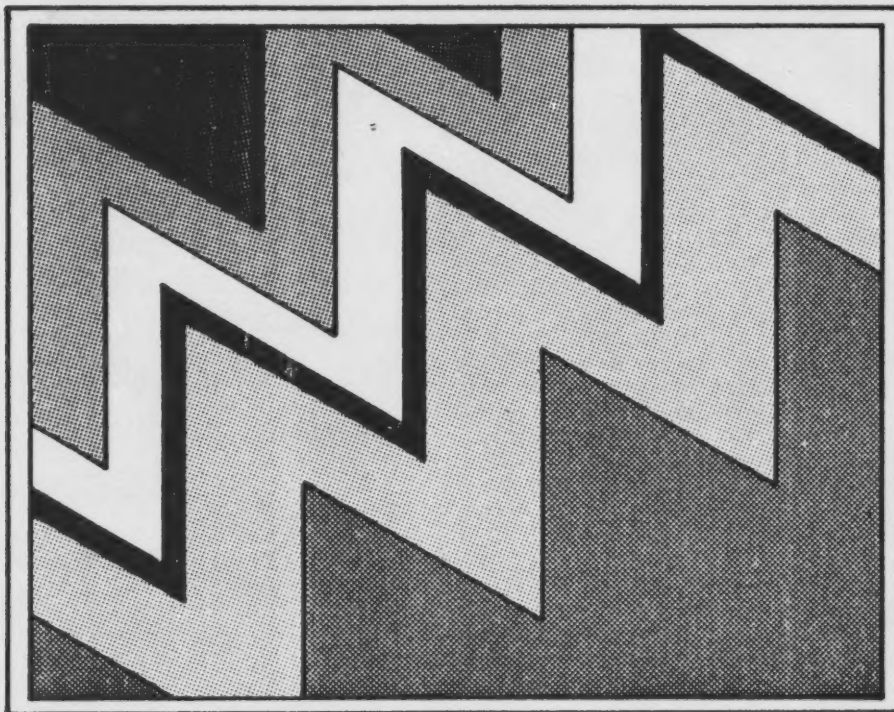
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UTC

UNIFIED TECHNICAL CONCEPTS

MODULE 7 E 1

ENERGY STORED
IN A BATTERY



ELECTRICAL APPLICATION OF
POTENTIAL AND KINETIC ENERGY

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

Operation of an electric storage battery involves transformation of energy from one form to another, subject to the principle of conservation of energy. Work required to charge the battery is greater than the resulting potential energy, which in turn is greater than the work the battery is able to do upon discharge. In both cases, some energy appears in unusable forms. Wherever storage batteries are needed to do electrical work, whether to power a pocket calculator or to start an automobile engine, it is important to understand that once the battery has done some work, sufficient energy can be restored in the battery so that it can do the work repeatedly.

PREREQUISITES

Student should have completed Concept Module 9-0, "Potential and Kinetic Energy."

OBJECTIVES

To complete this module, the student should be able to:

- Define the terms charge and potential difference and explain how they are related to energy.
- Draw schematic diagrams which show the circuit for both charging and discharging a battery on a resistive load, including the location of the necessary instruments and their terminal polarity.
- Given the necessary equipment, measure current and voltage of a storage battery during charge and discharge.
- Calculate the kinetic energy delivered by a battery during discharge, the electrical work required to recharge the battery and the potential energy of the charged battery.
- Calculate charge, potential energy, actual work done and work expended by a battery in a lawn mower system. All formulas and values are provided.

DISCUSSION

A battery is a chemical energy storage device. When it is used to do work, a chemical reaction releases energy through a circuit to an electrical appliance. As in the charging of a capacitor, accumulation of charge in a battery results in a change of potential difference across the terminals of the battery. However, the energy stored in a battery is maintained not by dielectric resistance but by the stability of the chemical materials in the battery cells.

Many kinds of electrochemical cells are used in batteries but they all have certain components in common, as shown in Figure 1: a positive electrode, a negative electrode and an electrolyte solution. Electrodes contain chemical compounds which react with the electrolyte during battery operation. There may be a separator between the positive and negative electrodes to prevent short circuits inside the cell. Each of the internal components provides some resistance to current flow so that during the charge or discharge of the battery, a definite internal resistance exists.

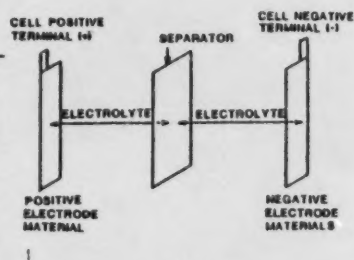


Fig. 1

When an automobile battery is used to turn a starter motor, an electrical circuit is completed between the terminals of the starter motor and the terminals of the battery (Figure 2). When the battery is not doing work, it is being recharged by the automobile generator (or alternator). The performance of a battery is tested by connecting an ammeter in series with the battery and a voltmeter in parallel with the battery (across the battery terminals), as shown.

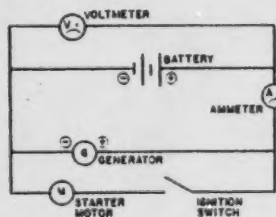


Fig. 2

Measurements of current and voltage can easily be translated into a description of electrical work. Recall from Concept Module 3-0 the general formula for the unified concept of work.

$$\text{Work} = \text{forcelike quantity} \times \text{displacementlike quantity} \quad (1)$$

and the special form of this equation for electrical energy:

$$\text{Work} = \text{potential difference} \times \text{charge moved.} \quad (2)$$

Potential difference is basically the forcelike effect

caused by a difference of charge accumulations between two points. When charge is moved, current flows and work is done. One joule of work is done when one coulomb of charge is moved through a potential difference of one volt. The potential energy of a battery can be described similarly. One joule of energy is stored if a cell has the ability to move one coulomb of charge through a potential difference of one volt. The conversion factors given in Table 1 show these relationships.

TABLE 1. CONVERSION FACTORS FOR ELECTRICAL WORK
(SI UNITS)

1 joule	= 1 volt x 1 coulomb.
1 coulomb	= electrical charge of 6.25×10^{18} electrons.
1 ampere	= 1 coulomb moved per second.
1 joule	= 1 volt x 1 ampere x 1 second.

To illustrate the comparison of work involved in charging and discharging a battery, consider the following example. A potential difference of 12.0 volts is measured at the terminals of a battery with no load. The battery is

then discharged at a constant rate of 20 amperes for one hour, voltages being recorded at the beginning of the discharge and at 15 minute intervals. The battery is then recharged at the same rate and voltages measured and recorded in the same manner. The results are tabulated in Table 2 and plotted in a graph (Figure 3).

TABLE 2. TYPICAL VOLTAGE MEASUREMENTS DURING DISCHARGE AND CHARGE OF A BATTERY

Time (minutes)	Discharge voltage (V)	Time (minutes)	Charge voltage (V)
0	11.5	0	11.8
15	11.2	15	12.1
30	11.1	30	12.2
45	11.1	45	12.2
60	11.0	60	12.3

When the data points for each operation are connected to form a curve, the area under each curve is proportional to the work done in that operation. A quick way to find the approximate area under a curve is to divide the graph into intervals on the horizontal axis, multiply the interval value by the average vertical value during that interval and then sum the products.

For this case, one can find the average voltage for each of the four fifteen minute intervals and multiply by the number of coulombs of charge moved at a rate of 20 amperes, using the conversion factors of Table 1.

$$\begin{aligned} \text{Charge moved} &= 20 \text{ amperes} \times \frac{1 \text{ coulomb}}{1 \text{ ampere} \times \text{second}} \\ &\times 15 \text{ minutes} \times \frac{60 \text{ seconds}}{1 \text{ minute}} = 18,000 \text{ coulombs.} \end{aligned}$$

The average voltages during the four 15 minute intervals are 11.35 V, 11.15 V, 11.10 V and 11.05 V. Using Equation 2, we multiply each voltage by 18,000 coulombs and we find that the work done during the one hour discharge operation is:

$$204,300 + 200,700 + 199,800 + 198,900 = 803,700 \text{ joules.}$$

A similar calculation for the four 15 minute intervals during charge shows that the average voltages are 11.95, 12.15, 12.20 and 12.25. The work required to charge the battery is then:

$$215,100 + 218,700 + 219,600 + 220,500 = 873,900 \text{ joules.}$$

The original potential energy of the battery at rest is its potential ability to discharge at 20 amperes for one hour.

$$\begin{aligned} \text{Potential energy} &= 12.0 \text{ V} \times 20 \text{ amperes} \times \frac{1 \text{ coulomb}}{1 \text{ ampere} \times \text{second}} \\ &\times 60 \text{ minutes} \times \frac{60 \text{ seconds}}{1 \text{ minute}} = 12.0 \text{ V} \times 72,000 \text{ coulombs} \\ &= 864,000 \text{ joules.} \end{aligned}$$

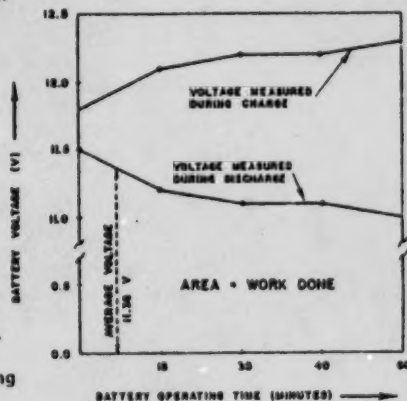


Fig. 3

Compare the work required to charge the battery to the potential energy of the battery at rest. What accounts for the apparent loss?

Compare the work done by the battery during discharge to the potential energy of the battery at rest. What accounts for the apparent loss?

Recall that a battery behaves in some ways like an electrical resistance. Thus, the movement of charge through the battery results in conversion of some of the kinetic energy into heat. Therefore, the potential energy of a charged battery is always less than the work required to charge it and the work which can be done by discharging a battery is always less than the potential energy of the battery before the discharge.

In the following laboratory experiment, voltages are measured during constant current discharging and recharging of a battery. For an interesting study in conservation of energy, perform the experiment three times, using a different value of current each time.

LABORATORY

MATERIALS

Rechargeable D-size cell battery.
Cell holder with terminal clips.
1-ohm resistor, power rated at 10 watts.
d.c. power supply (with current limiting capability) with a voltage range of at least 4 volts and a current range of at least one ampere.
d.c. ammeter with a range of at least one ampere. (If the power supply has a built-in ammeter, this will suffice.)
d.c. voltmeter with a full-scale range of at least 2.5 volts.
Eight pieces of 12 inch hook-up wire (4 red, 4 black). Use about 24 gage.
Terminal board with at least 8 positions.
Knife switch.
Timer with at least 1/2 minute resolution and 120 minute range.
Graph paper.

PROCEDURES

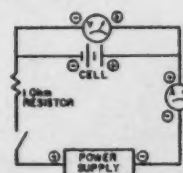
Follow Figure 4 as a model for the electrical network used in this experiment. Use red wire for connections to positive terminals of all components and black wire for connections to negative terminals.

1. Install cell in cell holder.
2. Connect voltmeter terminals to cell terminals (positive-to-positive and negative-to-negative).
3. Record voltage of cell at rest.

DISCHARGE PROCEDURE

4. Complete the electrical network indicated in Figure 4a. Ammeter terminals must be connected as shown - positive to cell positive and negative to power supply-negative. Check to be sure power supply is off and controls are set to minimum positions. Set timer to zero.
5. Close knife switch. Turn on power supply. Adjust current and voltage controls slightly above minimum and check ammeter and voltmeter to be sure both are indicating "positive." If not, turn off power supply and correct electrical connections as necessary.
6. Return current control to minimum and turn voltage control to maximum. Then increase current to 1.0 ampere. Start the timer.

(a) DISCHARGING CIRCUIT



(b) CHARGING CIRCUIT

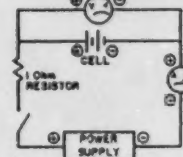


Fig. 4

7. Record the cell voltage measurement at about 15 seconds (call this $t = 0$) and at the 10 minute marks. When the voltage drops below 0.9 V, or at 60 minutes, whichever occurs first, record the voltage and discontinue operation.
8. Return controls to minimum, turn off power supply and open the knife switch.
9. Tabulate the data, using Table 2 as a model.

CHARGING PROCEDURE

10. Disconnect the cell-voltmeter assembly from the circuit, turn the assembly around so that the terminal polarities are opposite to the previous direction and remake the terminal connections. Follow the circuit shown in Figure 4b.
11. Check to be sure power supply is off and controls are set to minimum positions. Set timer to zero. Repeat steps 5 and 6 of the discharging procedure.
12. Record voltages the same way as in step 7. Notice that the voltage on charge will increase. Continue until charging time is equal to the time at which you discontinued the discharge. Record voltage at that point and discontinue operation.
13. Return controls to minimum, turn off power supply and open knife switch.
14. Tabulate the data as in step 9. Plot the data on graph paper, using Figure 3 as a model. Plot the discharge and charge data on the same graph.
Using Equation 2, the conversion factors of Table 1 and following the calculations given in the discussion, compute work and energy values from the experimental data.
15. Compute total charge moved during charge and discharge; this value is the same for both operations; namely,

$$\text{Charge moved (coulombs)} = \text{current (amperes)} \times \text{time (seconds)}.$$

(3)

Then, using Equation 2, compute potential energy of the open circuit cell.

16. For the discharge data, find the value of charge moved, average voltage and work done in each of the time segments. Sum the work values obtained for all time segments to get total work done by battery during discharge.
17. Repeat step 16, using charge data. Find total work required to charge battery.

18. Arrange results of step 16, 17 and 18 in increasing order. Compare their size and explain reason for the order.

SELF CHECK

1. Define the terms charge and potential difference.
2. Define electrical work in terms of charge and potential difference.
3. Draw a schematic diagram which shows the discharge of a battery on a resistive load. Include an ammeter and a voltmeter to show how electrical measurements are made during this operation. Show terminal polarities of the battery and meter.
4. Draw a schematic which shows a power supply being used to charge a battery. Include meters to show how electrical measurements are made. Show terminal polarities of the battery, the power supply and the meters.
5. A charged battery has an open circuit potential of 6.0 volts. It is used to power an electric lawn mower, which draws an average current of 5 amperes for 5 hours. Battery voltage during this time averages 5.0 volts. On a quick recharge, the average voltage is 7.0 volts during 90 minutes at 20 amperes. Compute the charge moved during the lawn mower operation, the potential energy of the charged battery available before discharge, the actual work done by the battery and the work expended in recharging the battery.

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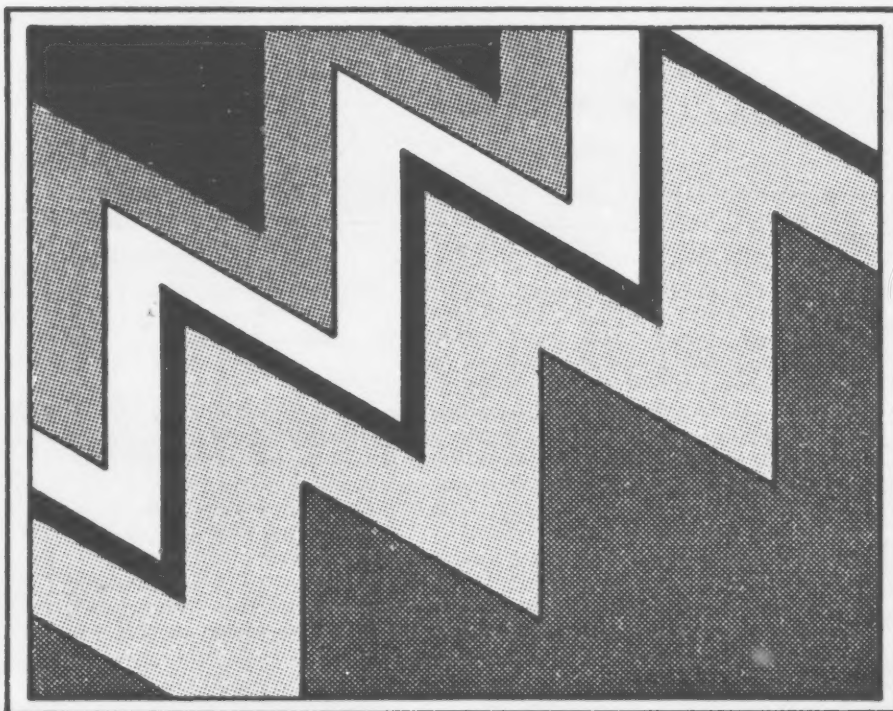
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UTC

UNIFIED TECHNICAL CONCEPTS

MODULE 7 E 2

ENERGY STORED IN A CAPACITOR



ELECTRICAL APPLICATION OF POTENTIAL AND KINETIC ENERGY

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

A capacitor is a device consisting of two closely-spaced conductors separated by a layer of insulating material, used to store electrical potential energy. This energy-storage capability is employed in many electronic circuits, including pulsed power supplies for lasers and other high-intensity light sources.

This module investigates the energy stored in a capacitor. In the laboratory, the student will measure such stored energy.

PREREQUISITES

Student should have completed Concept Module 7-0, "Potential and Kinetic Energy."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Calculate energy stored in a capacitor, given a known capacitance and voltage.
- Calculate voltage necessary to store a given energy in a capacitor of known capacitance.
- Determine the charge on each plate of a capacitor, given the size of the capacitor and the energy stored by it.
- Given an oscilloscope and other appropriate equipment, measure the energy stored in a capacitor.

DISCUSSION

A capacitor is a device used to store electrical potential energy. It usually consists of two large plates of conducting material separated by a layer of insulating material (dielectric). In one type of capacitor, the plates are made of metal foil, and the insulating layer is paper or plastic, allowing for the folding or rolling of the capacitor to attain a large plate area into a small package (Figure 1).

Capacitance, as defined by Equation 1, is a measure of the ability of a capacitor to store charge.

$$C = \frac{Q}{V} \quad (1)$$

where

C = capacitance.

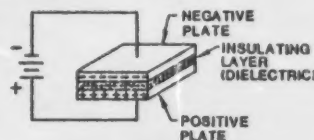
Q = charge stored.

V = voltage (potential difference).

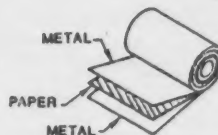
The unit of capacitance is the farad, equal to a coulomb of charge per volt of potential difference. Because a coulomb of charge is extraordinarily large, the farad is too large a unit of capacitance for practical applications. Capacitors are rated more frequently in microfarads (μF), where one microfarad equals 10^{-6} farads ($1 \mu F = 10^{-6} F$).

Figure 2 illustrates a capacitor in a simple charging circuit.

Assume that the capacitor is uncharged initially and that the switch is open (a). When the switch is closed (b), a current will flow to charge the capacitor. As more and more charge builds up on the capacitor, the transfer of additional charge becomes more and more difficult. Movement of charge stops when potential difference across the capacitor equals the potential difference of the battery. Work required to move the electrical charges is supplied by the battery and stored in the capacitor as electrical potential energy.



(a)



(b)

FIG. 1

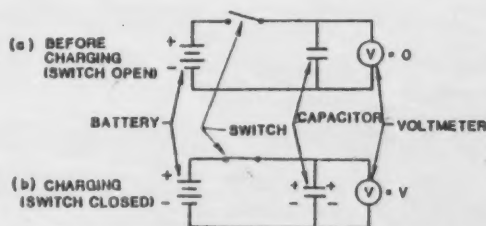


FIG. 2

The average potential difference through which the charge is moved is given by Equation 2.

$$V_{av} = \frac{V_{final} + V_{initial}}{2} = \frac{V + 0}{2} = \frac{1}{2} V \quad (2)$$

where

V_{av} = average voltage.

The total work done by the battery equals the energy stored in the capacitor; therefore, the energy stored is the product of the total charge transferred and the average voltage, as indicated by Equation 3.

$$\text{Work done} = \text{energy stored} = QV_{av} = Q \left(\frac{1}{2} V \right) = \frac{1}{2} QV \quad (3)$$

Substitution of $Q = CV$ from Equation 1 yields the frequently used form of Equation 4 for energy stored in a capacitor.

$$E_p = \frac{1}{2} CV^2 \quad (4)$$

where

E_p = energy stored in a capacitor.

When C is in farads and V in volts, E_p will be in joules.

Examples A and B demonstrate the use of Equation 4 in solving a problem.

EXAMPLE A: ENERGY STORED IN A CAPACITOR

Given: A $10\text{-}\mu\text{F}$ capacitor is charged with a voltage of 600 V.

Find: Amount of energy stored in the capacitor.

Solution: $E_p = \frac{1}{2} CV^2$
 $= \frac{1}{2} (10 \times 10^{-6} \text{ F}) (600 \text{ V})^2$ (Recall: $1 \mu\text{F} = 10^{-6} \text{ F}$.)
 $= (5 \times 10^{-6} \text{ F}) (3.6 \times 10^5 \text{ V}^2)$
 $= 1.8 \text{ FV}^2 = 1.8 \left(\frac{\text{coulomb}}{\text{volt}} \right) (\text{volt})^2$ (Note: A volt is a joule/coulomb.)
 $= 1.8 \left(\frac{\text{coulomb}}{\text{coulomb}} \right) \left(\frac{\text{joule}}{\text{coulomb}} \right)$
 $E_p = 1.8 \text{ joule}$

EXAMPLE B: VOLTAGE TO CHARGE A CAPACITOR

Given: 100 joules of energy must be stored in a 25- μ F capacitor.

Find: The necessary voltage.

Solution:

$$E_p = \frac{1}{2} CV^2$$

$$V^2 = \frac{2 E_p}{C}$$

$$V = \sqrt{\frac{2 E_p}{C}}$$

$$= \sqrt{\frac{2(100 \text{ J})}{25 \times 10^{-6} \text{ F}}}$$

$$= \sqrt{8 \times 10^6 \frac{\text{J}}{\text{F}}}$$

$$= 2.83 \times 10^3 \sqrt{\frac{\text{J}}{\text{F}}}$$

$$V = 2830 \text{ volts}$$

Note:

$$\sqrt{\frac{\text{J}}{\text{F}}} = \sqrt{\text{J} \times \frac{1}{\text{F}}}$$

$$= \sqrt{\text{joule} \times \left(\frac{\text{volt}}{\text{coulomb}}\right)}$$

$$= \sqrt{\left(\frac{\text{joule}}{\text{coulomb}}\right) \times \text{volt}}$$

$$= \sqrt{\text{volt}^2} = \text{volt}$$

SUMMARY

Capacitors are devices that store electrical potential energy. They consist of large plates of conducting material separated by layers of dielectric (insulating material). A popular type of capacitor is constructed of foil plates separated by paper or plastic, rolled into a convenient, small package. There are also large industrial capacitors, such as those employed on electrical power transmission equipment. Energy stored in capacitors is utilized in many circuits, such as pulsed power supplies and high-intensity light sources. They are used as "starters" on some electric motors and on fluorescent lights, where they deliver their stored energy in sudden bursts.

LABORATORY

ENERGY STORED BY CAPACITORS

MATERIALS

Oscilloscope with a triggered sweep.
 d.c. power supply (0-10 V, 50 W typical).
 Three capacitors (approximately 500 μF , 800 μF and 1000 μF).
 Assorted resistors (approximately 1 k Ω and 2 k Ω).
 Clip leads.
 Single-pole, double-throw switch.
 Graph paper (10 x 10).

PROCEDURE

1. Select a resistor, R, and a capacitor, C, that will give an RC time constant (R x C) of about one second. Note: (ohm) (farad) = $\frac{\text{volt} \times \text{second}}{\text{coulomb}}$
 $\frac{\text{coulomb}}{\text{volt}} = \text{second}.$
2. Connect equipment as shown in Figure 3.
3. Set oscilloscope controls in such a manner that an input of zero volts will result in a trace at lower edge of scope face and that d.c. voltage of power supply will give a trace near upper edge. Set trigger mode of oscilloscope to "line" and to "automatic." Start with time setting of 1 s/cm. This value may be changed later if necessary.
4. Throw switch to position A, as shown in Figure 3; observe scope. In five seconds (if RC was chosen to equal one second),

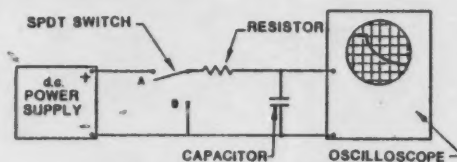


FIG. 3

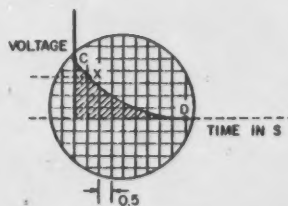


FIG. 4

capacitor should be almost fully charged. Throw switch to position B. Dot on scope should trace out a curve similar to that in Figure 4.

5. Point C in Figure 4 represents voltage across capacitor at the instant that switch is thrown to B position. This point is the beginning of capacitor discharge. Point D represents zero voltage across capacitor and is the point at which all charge has left the capacitor plates (about 5 seconds).
6. On the oscilloscope face, the horizontal portion of each square represents a certain time; and the vertical portion represents a certain voltage. These dimensions are indicated by the setting of oscilloscope knobs. For example, a knob setting of 0.5 s/cm means that horizontally each square represents 0.5 seconds. A knob setting of 2 volt/cm means that vertically each square represents 2 volts.
7. Graphing the oscilloscope trace. Throw switch to position A; after about 5 seconds, throw switch back to position B. Notice how many blocks the dot drops vertically as it moves one block horizontally. For example, if it drops one and a half blocks vertically, as in Figure 4, the point is 3-1/2 blocks above baseline; therefore, at 0.5 seconds, voltage across the capacitor is 7 volts (for a knob setting of 2 volt/cm). Enter voltage and time in Data Table 1. Also make a graph of voltage versus time. Repeat this procedure until enough points are chosen to make a good, smooth curve resembling Figure 4.

When horizontal axis is calibrated in time and vertical axis is calibrated in power, total energy stored in the capacitor and later dissipated by the resistor is represented by area under the trace curve (shaded area in Figure 4). To calibrate vertical axis as power, remember that power can be calculated from this equation:

$$P = \frac{V^2}{R}$$

where

P = power.

V = voltage.

R = resistance.

Power level at point C is determined from this equation if V represents the full voltage supplied.

8. Make several trials; determine approximate energy stored in capacitor by estimating area under the trace curve. (Refer to Example C.) Record results in Data Table 2.

9. Repeat process with two other capacitors.
10. Calculate energy stored in each capacitor; use $E_p = \frac{1}{2} CV^2$.
11. Compare calculated results with estimated energy stored.
12. Discuss the importance of capacitors as energy-storage devices.

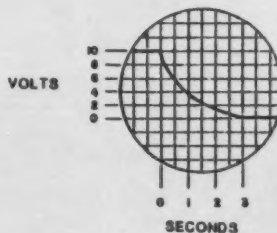
EXAMPLE C: ESTIMATING ENERGY STORED IN A CAPACITOR

Given: Oscilloscope trace similar to figure on right.

Find: Energy sotred in capacitor.

Solution: Assume the following values:

$C = 500 \mu\text{F}$ Note:
 $R = 2\text{k}\Omega$ $RC = 1 \text{ s}$
 $V = 10 \text{ V}$



Consider the first time interval of one second.

Instantaneous voltage at start = 10 volts.

Instantaneous voltage at end = 3.7 volts.

$$\text{Average voltage over first second interval} = \frac{10 + 3.7}{2}$$

$$= 6.85 \text{ volts}$$

$$\text{Average power over first second interval} = \frac{(6.85)^2}{2000}$$

$$= 0.0235 \text{ watts}$$

Decrease in potential energy of capacitor over first second interval = $\bar{P}\Delta t = (0.0235 \text{ joules/s} \times 1 \text{ s}) = 0.0235 \text{ joules}$.

Time (seconds)	Instantaneous Voltage, V (volts)	Average Voltage, $\bar{V}_{av}$ (volts)	Average Power, $\bar{P}_{av} = \frac{\bar{V}^2}{R}$ (watts)	Energy, $\bar{P}\Delta t$ (joules)
0	10	--		
1	3.7	6.85	0.0235	0.0235
2	1.4	2.55	0.0033	0.0033
3	0.5	0.95	0.0005	0.0005

Therefore, total energy stored in the capacitor can be estimated to be $0.0235 + 0.0033 + 0.0006$ joules = $0.027 \text{ J} = 27 \times 10^{-3} \text{ J}$.

Use assumed values in Equation 4:

$$\begin{aligned} E_p &= \frac{1}{2} CV^2 \\ &= \frac{1}{2} (0.5 \times 10^{-3} \text{ F})(100 \text{ V}^2) \\ E_p &= 25 \times 10^{-3} \text{ J} \end{aligned}$$

This solution compares favorably with the estimated value.

DATA TABLE 1	
Voltage	Time

DATA TABLE 2					
Capacitance	Resistance	Voltage	Estimated Energy (Area under Curve, Example C)	Calculated Energy ($E_p = \frac{1}{2}CV^2$)	% Difference
500 μF					
800 μF					
1000 μF					

SELF CHECK

1. A $2000\text{-}\mu\text{F}$ capacitor is charged to 240 V . How much energy is stored?
2. A $375\text{-}\mu\text{F}$ capacitor is charged to 4000 V . How much energy is stored?
3. A $400\text{-}\mu\text{F}$ capacitor is to be used to store one kilojoule. What will the voltage be?
4. A $0.25\text{-}\mu\text{F}$ capacitor is to be used to store 72 J . What will the voltage be?
5. 200 J is stored in a $60\text{-}\mu\text{F}$ capacitor. What is charge on each plate of the capacitor?

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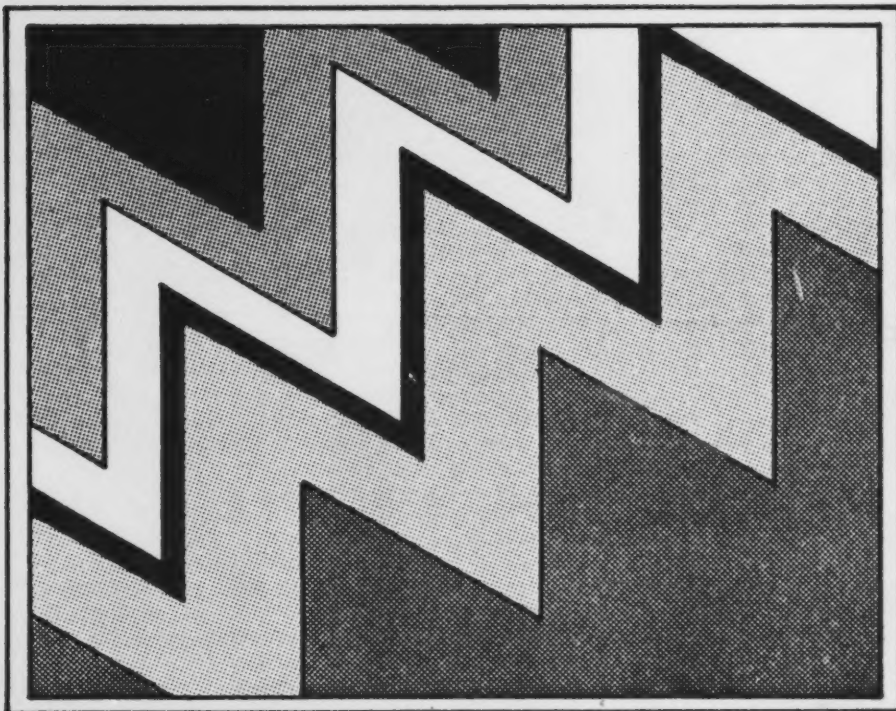
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UTC

UNIFIED TECHNICAL CONCEPTS

MODULE 7 E 3

ENERGY STORED IN
AN INDUCTOR



ELECTRICAL APPLICATION OF POTENTIAL AND KINETIC ENERGY

Technical Education Research Center - Southwest

ERC - SW

INTRODUCTION

Energy supplied to an electric circuit is dissipated by its resistance and is stored in its electric and magnetic fields. Electrical potential energy is stored in the electric field between the conductors of a capacitor. Magnetic potential energy is stored in the magnetic field of an inductor.

The concept of inductance is the subject of this module. Air-core and iron-core inductors are contrasted. The reaction of the inductor to changing currents is discussed in terms of energy stored in its magnetic field. In the laboratory, the student will measure the current through an inductor and will calculate the energy stored in its magnetic field.

PREREQUISITES

The student should have completed Concept Module 7-0, "Potential and Kinetic Energy."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define "inductance."
- Calculate the inductance and magnetic-field strength of an air-core inductor.
- State the difference in inductance and magnetic field strength of a coil with and without an iron core.
- Describe how a variable inductor works.
- Calculate the energy stored in the magnetic field of an inductor, given the inductance and current.
- Sketch graphs of voltage, current, and energy stored as functions of time in an inductor through which the current is increasing and decreasing.

- Sketch a schematic of the ignition system of an automobile and describe how it produces a spark; include the functions of the coil, breaker points, condenser, and rotor.
- Given the appropriate equipment, measure the current through an inductor of known inductance; and calculate the energy stored in its magnetic field.

DISCUSSION

A varying magnetic field induces an electromotive force, or emf, in a conducting loop within the field (Figure 1). The emf is in such a direction as to oppose, by the magnetic action of a corresponding induced current, the changes in the magnetic field linking the loop (Lenz's Law); however, the induced emf occurs whether or not there is current in the loop. If the magnetic field is itself due to a current in the loop, then the induced emf will act in such a way as to resist changes in the magnetic field or changes in the loop current responsible for the magnetic-field changes. A current loop, thus, tends to preserve the status-quo through a kind of built-in inertia. This resistance to changing current is the inductance of the loop.

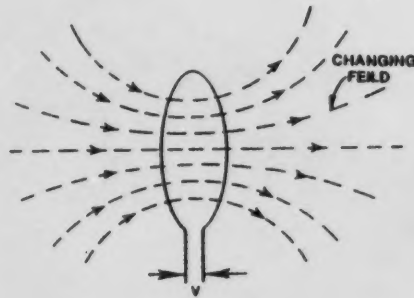


FIG. 1

If the loop is a closed circuit, an induced current accompanies the induced emf. It circulates in such a manner as to oppose the changes in the applied current. When the applied current and associated magnetic field are constant, the induced emf and current disappear.

Mathematically, Inductance is defined as follows by Equation 1.

$$L = \frac{-V}{\left(\frac{\Delta I}{\Delta t}\right)} \quad (1)$$

where

L = inductance in Henries.

V = induced emf.

ΔI = average change in current.

Δt = time interval in which current change occurs.

The minus sign appears because the emf V opposes the change in current ΔI . The unit of inductance in the International System of units is the Henry, abbreviated H. From Equation 1, the Henry is defined as follows:

$$1 \text{ Henry} = 1 \text{ volt}/(1 \text{ amp}/1\text{s})$$

$$= 1 \text{ (volt-s/amp)}$$

$$\text{or } 1 \text{ Henry} = 1 \Omega\text{-s (since } 1 \Omega = 1 \text{ volt/amp)}$$

Inductance in a circuit is represented in the schematic form of a coil, as illustrated in Figure 2. The common inductor is a coil of wire. The Inductance of an air-core coil is given by Equation 2.

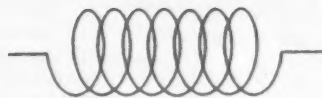


FIG. 2

$$L = \mu_0 N^2 \frac{A}{\ell} \quad (2)$$

where

- L = inductance of the coil.
- μ_0 = constant ($4\pi \times 10^{-7}$ H/m), called the "permeability of free space."
- N = number of loops or turns of the coil.
- A = cross-sectional area.
- ℓ = length.

The magnetic field strength of an air core inductor is —

$$B = \mu_0 \frac{NI}{\ell} \quad (3)$$

where

- B = magnetic field strength, in Teslas.
- I = current.

The unit of B is the Tesla, abbreviated T. The magnetic field is parallel to the axis of the coil, as indicated by Figure 3.

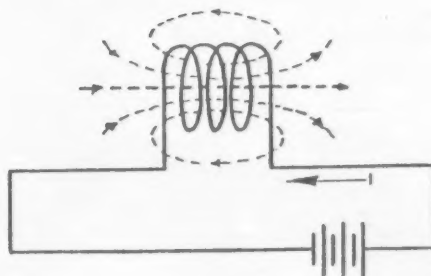


FIG. 3

The inductance and magnetic field strength of the coil are enhanced if the windings are wrapped on an iron core (Figure 4a). The inductance and magnetic field strength now are given by Equations 4 and 5.

$$L = \mu_i N^2 \frac{A}{\ell} \quad (4)$$

$$\text{and } B = \mu_i \frac{NI}{\ell} \quad (5)$$

where

- μ_i = permeability of iron.

The permeability of free space μ_0 has been replaced by the permeability of iron μ_i . The permeability of iron is not constant, but depends upon the current I through the coil and the history of the iron sample. The permeability of a material, therefore, is a measure of how easily and to what extent it can be magnetized. The permeability of iron is quite large. Once magnetized, iron will preserve a strong magnetic field, unless environmental factors destroy it. Because of this preservation, it is said to form a "permanent" magnet. However, its permeability changes as the external magnetic field changes; this phenomenon is known as "hysteresis."

A variable inductor is depicted in Figure 4b. As the iron rod slides into the coil, the effective permeability is due more to iron than to air. The inductance, therefore, increases as more of the rod slides inside the coil. Example A shows how the inductance and magnetic-field strength of an inductor are calculated.

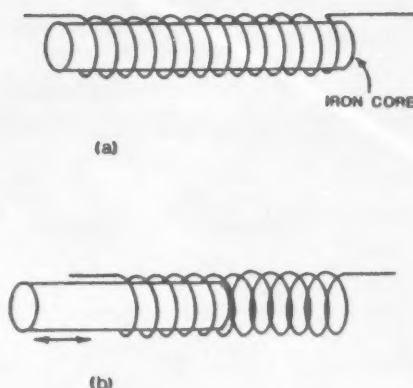


FIG. 4

EXAMPLE A: AIR CORE INDUCTOR

Given: An air-core inductor has $N = 1000$ turns, $l = 2$ cm, and a radius $r = 2$ cm. The current $I = 1.0$ A.

Find: (a) The inductance L of the coil.
(b) The magnetic-field strength B at the center of the coil.

Solution: (a) $A = \pi r^2$
 $= \pi (0.02 \text{ m})^2$
 $A = 0.00125 \text{ m}^2$
 $L = \mu_0 \frac{N^2 A}{l}$
 $= \frac{(4\pi \times 10^{-7} \text{ H/m}) (1000)^2 (0.00125 \text{ m}^2)}{(0.02 \text{ m})}$
 $L = 78.5 \text{ mH}$
 (b) $B = \mu_0 \frac{NI}{l}$
 $= \frac{(4\pi \times 10^{-7} \text{ H/m}) (1000) (1.0 \text{ A})}{(0.02 \text{ m})}$
 $B = 0.0628 \text{ T}$

ENERGY STORED IN AN INDUCTOR

As long as the current through the inductor is changing, there is an emf developed along the coil; therefore, power (recall: $P = VI$) or energy is supplied to the inductor from the circuit. If the current becomes constant, there is no induced emf and, therefore, zero power input. The energy supplied to the inductor is used to establish its magnetic field. This energy remains stored as magnetic potential energy as long as the current remains constant. If the circuit is opened, the magnetic field collapses; and the energy is returned to the circuit. The release of energy often is sufficient to create an arc across the switch that opens the circuit.

When the current in a circuit increases from zero to I , the energy supplied to the inductor can be calculated from Equation 6.

$$E = \frac{1}{2} LI^2 \quad (6)$$

where

E = energy.

When a voltage V is applied suddenly to an inductor, as when switch S_1 is closed in the circuit of Figure 5, the current in the circuit increases only gradually, because of the coil's resistance to change in the current. Thus, time is required for the energy supplied to the inductor from the battery to be stored in the magnetic field. Voltage across the inductor leads the current, but the time lapse between voltage and current typically changes only a few milliseconds. Graphs of voltage, current, and energy as functions of time are shown in Figure 6.

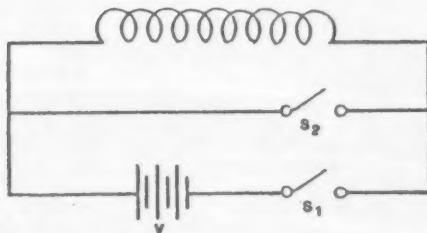


FIG. 5

Likewise, if the applied voltage drops suddenly to zero, as when switch S_2 of Figure 5 is closed, time is required for the energy stored in the magnetic field of the inductor to be released to the circuit. The current decays exponentially to zero. The current change lags the voltage change. Graphs of voltage, current, and energy as functions of time during current decay are shown in Figure 7. Calculation of the energy stored in an inductor is illustrated in Example 8.

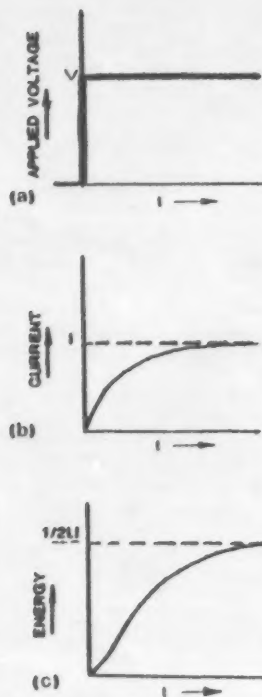


FIG. 6

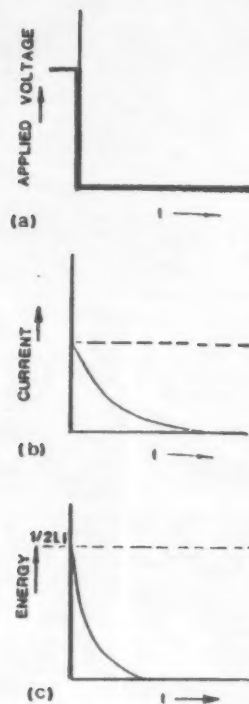


FIG. 7

EXAMPLE B: ENERGY STORED IN AN INDUCTOR	
<u>Given:</u>	The current I through an air-core inductor of inductance $L = 1.0 \text{ H}$ is 2.0 A .
<u>Find:</u>	The energy stored in the magnetic field of the inductor.
<u>Solution:</u>	$E = \frac{1}{2} LI^2$ $= \frac{1}{2} (1.0 \text{ H})(2.0 \text{ A})^2$ $E = 2.0 \text{ joules}$

THE IGNITION SYSTEM OF AN AUTOMOBILE

Figure 8 is a schematic diagram of an automobile ignition system. It consists of two separate circuits. The primary cir-

circuit is composed of the battery, ignition switch, primary winding of the ignition coil, breaker points, and condenser. The secondary circuit contains the secondary winding of the ignition coil, rotor, and spark plugs. The breaker points, condenser, and rotor are contained in the distributor.

The primary winding typically has several hundred turns of wire, while the secondary one has more than 10,000; therefore, the inductance of the secondary is very large compared to that of the primary. When the current through the primary changes, a large emf is induced in the secondary.

The process that results in the spark begins with the ignition switch and breaker points closed, establishing a constant current through the primary circuit. The breaker points are opened by a revolving cam. The rapidly-decreasing current in the primary induces a large emf in the secondary. The rotor is a rotary switch that connects the secondary to one of the spark plugs. The emf in the secondary is discharged across the gap in the spark plug, igniting the fuel in the cylinder. The process requires precise timing, since the breaker points are open for less than five thousandths of a second for each spark; and the spark must occur at the top of the compression cycle in the cylinder. The condenser is a capacitor that prevents sparking across the breaker points when they are open.

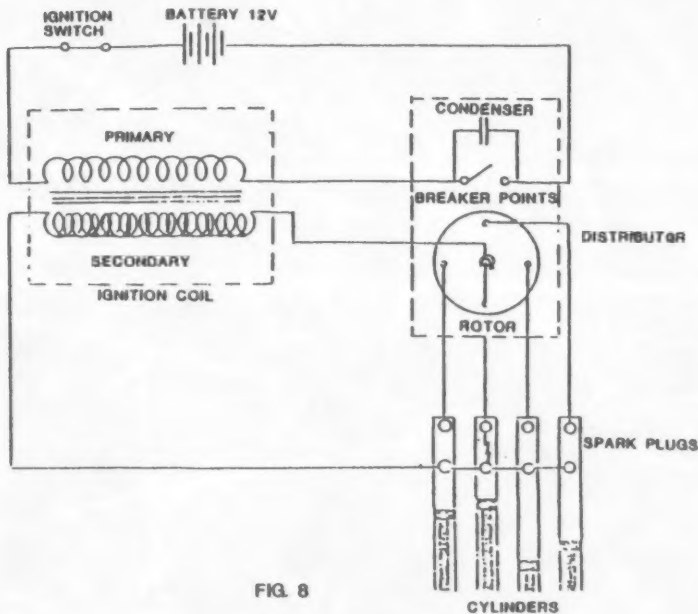


FIG. 8

SUMMARY

The inductor is a device that resists changes in the current passing through it. The energy stored in the magnetic field of an inductor is derived by the following equation:

$$E = \frac{1}{2} LI^2$$

where

E = energy.

L = inductance.

I = current.

An iron-core inductor has a larger inductance and, therefore, is capable of storing more magnetic energy than the same coil with an air core. The large induced emf of a high-inductance coil produces the spark that ignites the fuel in the cylinder of an automobile engine.

LABORATORY

DETERMINATION OF ENERGY STORED IN AN INDUCTOR

MATERIALS

Automobile ignition coil.
d.c. power supply (0-12 V).
Switch.
2 μ F capacitor.
Ammeter.
Spark plug.

PROCEDURE

1. Wire the schematic shown in Figure 9. Close switch, and adjust power supply for an output voltage of 12 volts.
2. In the Data Table, record the current registered on the ammeter.
3. Calculate energy stored in magnetic field of primary, using $L = 4\text{H}$ and measured current; and record in results portion of Data Table.
4. The following procedure must be done with extreme caution because voltage developed across secondary is typically 10,000 to 20,000 volts. Open switch, and observe spark across spark plug.
5. Repeat steps 2, 3, and 4 for output voltages of 10, 8, 6, and 4 volts. Record values.

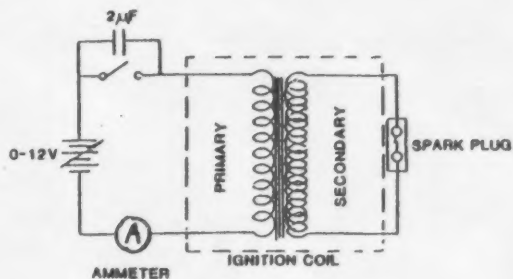


FIG. 9

DATA TABLE		
		RESULTS
Output Voltage (volts)	Primary Current (amperes)	Energy $E = \frac{1}{2} LI^2$ (joules)
12		
10		
8		
6		
4		

SELF CHECK

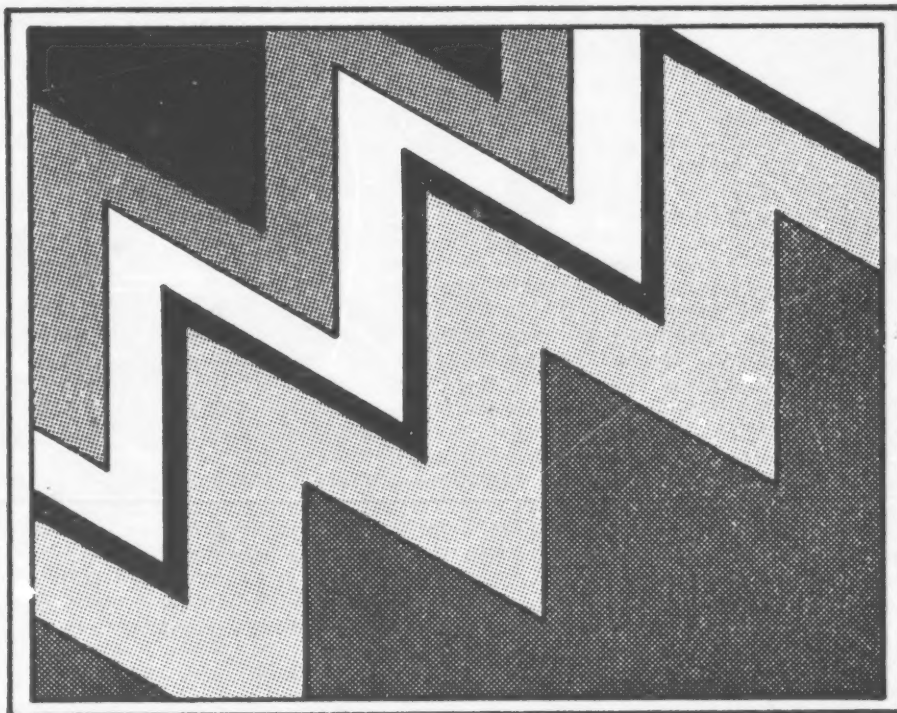
1. Define "inductance."
2. An air core inductor has $N = 3500$ turns, $l = 7.0$ cm, and $r = 5.0$ cm. Calculate the following values:
 - a. Inductance of the coil.
 - b. Magnetic-field strength for a current of 0.5 A.
 - c. Energy stored in the magnetic field of the coil for a current of 0.5 A.
3. State the differences in inductance and magnetic field strength of a coil with and without an iron core.
4. Describe how a variable inductor works.
5. Sketch graphs showing the time dependence of voltage, current, and energy for an inductor when the current increases and when it decreases.
6. Sketch a schematic of an automobile ignition system, and describe how it produces a spark. Include the functions of the coil, breaker points, condenser, rotor, and spark plugs.

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ELECTRICAL APPLICATION OF POTENTIAL AND KINETIC ENERGY

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

Often, circuits in which capacitors are arranged in combinations must be designed or analyzed. When the combinations are simple series or parallel networks, the analysis is particularly simple.

In this module, the student will become familiar with the properties of circuits consisting of capacitors connected in series or parallel. The concept of the potential energy of equivalent capacitance will be introduced, and practical applications will be discussed briefly. In the laboratory, the student will verify these properties and will apply them to measure an unknown capacitance.

PREREQUISITES

Student should have completed Concept Module 7-0, "Potential and Kinetic Energy," and Application Module 7E2, "Energy Stored in a Capacitor."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Calculate the equivalent capacitance of a series or parallel network of capacitors.
- Given two of the following three quantities, calculate the third:

Equivalent capacitance.
Source voltage.
Charge.

- Given two of these three quantities, calculate the third:

Equivalent capacitance.
Source voltage.
Energy.

- List four properties of equivalent circuits for –

Parallel-connected capacitors.

Series-connected capacitors.

- Given the appropriate equipment, measure the voltage across capacitors connected in series or parallel, and calculate the charge and energy stored by each.
- Given the appropriate equipment, determine the capacitance of an unknown capacitor.

DISCUSSION

CAPACITORS IN PARALLEL

Figure 1 is a schematic of two capacitors, C_1 and C_2 , connected in parallel across a d.c. voltage source V . Because the capacitors are in parallel, the voltage across each must be the same as, and equal to, V . Hence it is a simple matter to calculate the magnitude of the charge carried on the plates of each capacitor. Equation 1 from Module 7E2 is as follows:

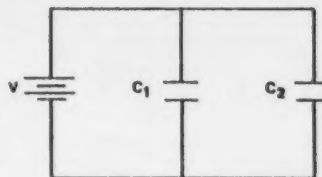


FIG. 1

$$C = \frac{Q}{V} \quad (1)$$

where

C = capacitance.

Q = charge stored.

V = voltage (potential difference).

When both sides of Equation 1 are multiplied by V , the result is Equation 2.

$$Q = CV \quad (2)$$

According to Equation 2, the charge Q_1 on capacitor C_1 in Figure 1 is equal to C_1V , and the charge Q_2 on C_2 is C_2V . In a similar manner, the energy stored by each capacitor can be calculated using Equation 4 from Module 7E2:

$$E_p = \frac{1}{2} CV^2 \quad (3)$$

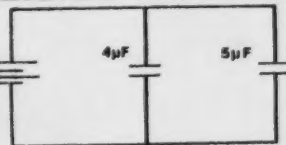
where

E_p = potential energy stored in a capacitor.

Equation 3 indicates that the energy E_1 stored by C_1 is equal to $1/2 C_1V^2$, and the energy E_2 stored by C_2 is $1/2 C_2V^2$. Example A illustrates how Equations 2 and 3 can be applied to solve a problem.

EXAMPLE A: CHARGE AND ENERGY STORED BY CAPACITORS IN PARALLEL

Given: A $4\text{-}\mu\text{F}$ and a $5\text{-}\mu\text{F}$ capacitor are connected in parallel across a 20-V d.c. source.



Find: (a) The charge stored by each capacitor.
(b) The energy stored by each capacitor.

Solution: (a) $Q = CV$ (Equation 2)

$4\text{-}\mu\text{F}$ capacitor:

$$\begin{aligned} Q &= (4 \times 10^{-6} \text{ F})(20 \text{ V}) \\ &= (80 \times 10^{-6} \text{ F})(\text{V}) \\ Q &= 80 \times 10^{-6} \text{ C} \end{aligned}$$

$5\text{-}\mu\text{F}$ capacitor:

$$\begin{aligned} Q &= (5 \times 10^{-6} \text{ F})(20 \text{ V}) \\ &= (100 \times 10^{-6} \text{ F})(\text{V}) \\ Q &= 100 \times 10^{-6} \text{ C} \end{aligned}$$

Note: $1 \text{ FV} = 1 \frac{\text{C}}{\text{V}} \text{ V}$

$1 \text{ FV} = 1 \text{ C (1 coulomb)}$

(b) $E_p = \frac{1}{2} CV^2$ (Equation 3)

$4\text{-}\mu\text{F}$ capacitor:

$$\begin{aligned} E_p &= \left(\frac{1}{2}\right)(4 \times 10^{-6} \text{ F})(20 \text{ V})^2 \\ &= (800 \times 10^{-6} \text{ F})(\text{V}^2) \\ E_p &= 800 \times 10^{-6} \text{ J} \end{aligned}$$

$5\text{-}\mu\text{F}$ capacitor:

$$\begin{aligned} E_p &= \left(\frac{1}{2}\right)(5 \times 10^{-6} \text{ F})(20 \text{ V})^2 \\ &= (1000 \times 10^{-6} \text{ F})(\text{V}^2) \\ E_p &= 1000 \times 10^{-6} \text{ J} \end{aligned}$$

Note: $1 \text{ FV}^2 = 1 \frac{\text{C}}{\text{V}} \text{ V}^2$

$= 1 \text{ CV}$

$= 1 \text{ C } \left(\frac{\text{J}}{\text{C}}\right)$

$1 \text{ FV}^2 = 1 \text{ J (1 joule)}$

A useful concept often used in analyzing combinations of capacitors in a circuit is "equivalent capacitance." The equivalent capacitance of a group of capacitors in a circuit is defined as "the capacitance of the single capacitor that can replace the group without changing the characteristics of the circuit." A circuit in which two or more capacitors are replaced by their equivalent capacitance is called an "equivalent circuit." For the case in which the original group of capacitors are connected in parallel, the equivalent capacitance C_p easily is found.

Figure 2a depicts the circuit previously shown and indicates the charge and energy associated with each capacitor. Since the characteristics of the equivalent circuit must be identical to those of the original circuit, the total potential energy E_p stored by C_p must be equal to the sum of the energies stored by C_1 and C_2 . Similarly, the total charge Q_T on C_p must equal to the sum of the charges on C_1 and C_2 (Figure 2b). Either of these criteria can be used to determine the equivalent capacitance. Using the latter:

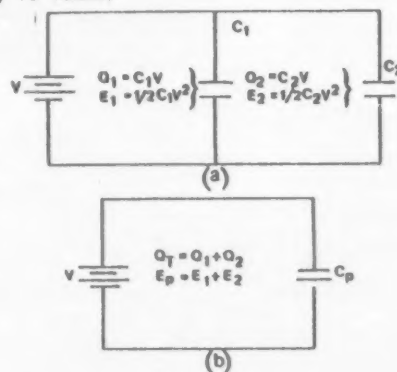


FIG. 2

$$Q_T = Q_1 + Q_2$$

$$C_p V = C_1 V + C_2 V \quad (\text{substitution from Equation 2})$$

$$C_p = C_1 + C_2 \quad (\text{division of both sides by } V)$$

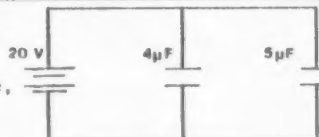
As seen above, the equivalent capacitance of C_1 and C_2 connected in parallel is exactly the sum of their individual capacitance. In general, for any number of capacitors connected in parallel, the equivalent capacitance is simply the sum of the individual capacitances, as indicated by Equation 4.

$$C_p = C_1 + C_2 + C_3 + \dots \text{etc.} \quad (4)$$

In addition, the total charge and energy stored by the circuit can be calculated directly from Equations 2 and 3, respectively, by using the value of C_p for the capacitance.

EXAMPLE B: EQUIVALENT CAPACITANCE OF A PARALLEL NETWORK

Given: A $4\text{-}\mu\text{F}$ and a $5\text{-}\mu\text{F}$ capacitor are connected in parallel across a 20-V d.c. voltage source, as in Example A.



Find: (a) Their equivalent capacitance.
(b) The total charge stored.
(c) The total energy stored.

Solution: (a) $C_p = C_1 + C_2$
 $= 4\mu\text{F} + 5\mu\text{F}$
 $C_p = 9\mu\text{F}$

(b) $Q_T = C_p V$
 $= 9 \times 10^{-6} \text{ F} (20 \text{ V})$
 $= 180 \times 10^{-6} \text{ C}$
 $Q_T = 180\mu\text{C}$

Note that this is the sum of Q_1 and Q_2 from Example A.

(c) $E_p = \frac{1}{2} C_p V^2$
 $= \frac{1}{2} (9 \times 10^{-6} \text{ F}) (20 \text{ V})^2$
 $E_p = 1800 \times 10^{-6} \text{ J}$

Note that this value is the sum of E_1 and E_2 from Example A.

The circuit behaves exactly as if a single $9\text{-}\mu\text{F}$ capacitor were connected across the voltage source.

In summary, the equivalent circuit of a group of capacitors connected in parallel exhibits the following properties:

1. The equivalent capacitance is equal to the sum of the individual capacitances:

$$C_p = C_1 + C_2 + C_3 + \dots \text{ etc.} \quad (4)$$

2. The voltage across each individual capacitor is the same as, and equal to, the total voltage V_T across the equivalent capacitor:

$$V_T = V_1 = V_2 = V_3 = \dots \text{ etc.} \quad (5)$$

3. The total charge Q_T associated with the equivalent capacitor is equal to the sum of the charges of the individual capacitors:

$$Q_T = Q_1 + Q_2 + Q_3 + \dots \text{ etc.} \quad (6)$$

4. The total potential energy E_p stored by the equivalent capacitor is equal to the sum of the energies stored by each individual capacitor:

$$E_p = E_1 + E_2 + E_3 + \dots \text{ etc.} \quad (7)$$

The principal application of capacitors connected in parallel is in the building of large capacitance networks from smaller capacitors in parallel. This procedure results in high capacitance values at less expenditure than is required for a single large capacitor.

CAPACITORS IN SERIES

Figure 3a displays capacitors C_1 and C_2 connected in series across potential difference V . In general, the voltages V_1 and V_2 across each capacitor will be different — unlike the case for capacitors in parallel; however, the charge that accumulates on the plates of each capacitor is the same.

Figure 3b shows the circuit as the capacitors are beginning to charge. The process can be visualized step-by-step as follows: A negative charge (an electron) leaves the negative terminal of the battery and travels to the nearest plate of C_2 , imparting to it a slight, negative net charge (Figure 3b). Because the charge on one plate of a capacitor induces an equal and opposite charge on the opposite plate, an electron leaves the opposite plate of C_2 and travels to the nearest plate of C_1 (Figure 3c). Since the plates of each capacitor at first are electrically neutral, a positive charge of equal magnitude is left behind on C_2 . Upon arriving at C_1 , the electron repels another electron from the opposite plate; this electron then moves to the positive terminal of the battery (Figure 3d). The process continues until the capacitors are charged completely, each carrying equal charges Q (Figure 3e). Notice that although each capacitor carries a charge

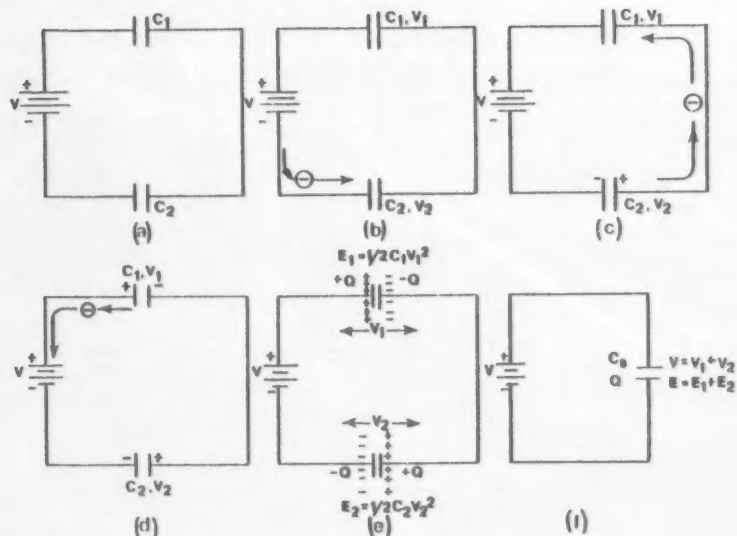


FIG. 3

equal to Q , the battery has only supplied a total charge Q_T to the entire circuit. The net effect, then, is that of a single capacitor carrying a charge Q . Thus, the charge Q_T carried by the equivalent capacitor should be the same as the charge on each of the individual capacitors.

In order that the equivalent circuit can behave in the same manner as the series circuit, two more criteria must be met: (1) The energy stored by the equivalent circuit must be equal to the sum of the energies stored in the individual capacitors. (2) The total voltage across the equivalent circuit must be equal to the sum of the voltages across the individual capacitors. Either of these criteria (shown in the equivalent circuit of Figure 3f) can be used to determine the value of the series equivalent capacitance, C_S . Equation 8 derives the total voltage.

$$V_T = V_1 + V_2 \quad (8)$$

Rearranged, Equation 1 gives $V = Q/C$. Substitution of the appropriate value of V and C in this result transforms Equation 8 into —

$$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} \quad (\text{Both sides divided by } Q)$$

Thus, the reciprocal of the equivalent capacitance of the two capacitors connected in series is equal to the sum of the reciprocals of the individual capacitances. This relationship is true in general, no matter how many capacitors are connected in series; therefore —

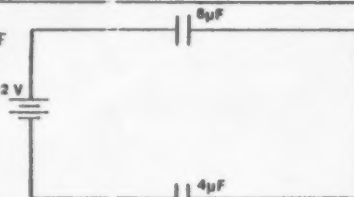
$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots \text{ etc.} \quad (9)$$

Once C_s is known, the charge and energy stored by the equivalent circuit can be calculated from Equations 2 and 3 respectively, using the value of C_s for the capacitance. Example C demonstrates the solution of a typical problem involving capacitors in series.

EXAMPLE C: SERIES CAPACITOR CIRCUIT

Given: A $6\text{-}\mu\text{F}$ capacitor and a $4\text{-}\mu\text{F}$ capacitor are connected in series across a 12-V d.c. voltage source.

- Find:**
- The equivalent capacitance of the circuit.
 - The total charge supplied by the battery.
 - The total energy stored in the circuit.
 - The voltage across each capacitor.
 - The energy stored in each capacitor.



Solution:

$$\begin{aligned} \text{(a)} \quad \frac{1}{C_s} &= \frac{1}{C_1} + \frac{1}{C_2} \\ &= \frac{1}{6\mu\text{F}} + \frac{1}{4\mu\text{F}} \\ &= \frac{2}{12\mu\text{F}} + \frac{3}{12\mu\text{F}} \quad (\text{Expressed in terms of common denominators}) \\ &= \frac{5}{12\mu\text{F}} \quad (\text{The fractions are added.}) \\ \frac{1}{C_s} &= \frac{12\mu\text{F}}{5} \quad (\text{The reciprocal of both sides is taken.}) \\ C_s &= 2.4\mu\text{F} \\ \text{(b)} \quad Q_T &= C_s V \\ &= 2.4 \times 10^{-6} \text{ F (12 V)} \\ Q_T &= 28.8 \times 10^{-6} \text{ C} \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad E_P &= \frac{1}{2} C_S V^2 \\
 &= \frac{1}{2} (2.4 \times 10^{-6} \text{ F}) (12 \text{ V})^2 \\
 E_P &= 172.8 \times 10^{-6} \text{ J}
 \end{aligned}$$

(d) 6- μ F capacitor:

$$\begin{aligned}
 V &= \frac{Q}{C} \\
 &= \frac{28.8 \times 10^{-6} \text{ C}}{6 \times 10^{-6} \text{ F}} \\
 V &= 4.8 \text{ V}
 \end{aligned}$$

4- μ F capacitor:

$$\begin{aligned}
 V &= \frac{28.8 \times 10^{-6} \text{ C}}{4 \times 10^{-6} \text{ F}} \\
 V &= 7.2 \text{ V}
 \end{aligned}$$

Notice that the sum of the voltages across the individual capacitors is equal to the voltage across the equivalent capacitor (source voltage):

$$4.8 \text{ V} + 7.2 \text{ V} = 12 \text{ V}$$

$$\text{(e)} \quad E_P = \frac{1}{2} CV^2$$

6- μ F capacitor:

$$\begin{aligned}
 E_P &= \frac{1}{2} (6 \times 10^{-6} \text{ F}) (4.8 \text{ V})^2 \\
 E_P &= 69.12 \times 10^{-6} \text{ J}
 \end{aligned}$$

4- μ F capacitor:

$$\begin{aligned}
 E_P &= \frac{1}{2} (4 \times 10^{-6} \text{ F}) (7.2 \text{ V})^2 \\
 E_P &= 103.68 \times 10^{-6} \text{ J}
 \end{aligned}$$

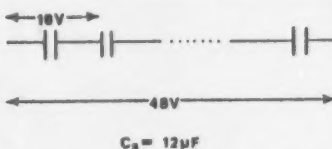
Notice that the sum of the energies stored by each capacitor is equal to the energy stored by the equivalent capacitor:

$$69.12 \times 10^{-6} \text{ J} + 103.68 \times 10^{-6} \text{ J} = 172.8 \times 10^{-6} \text{ J}$$

Example D illustrates another application of capacitors in series.

EXAMPLE D: ENHANCED VOLTAGE RATING OF CAPACITORS IN SERIES

Given: A technician must construct a 12- μF equivalent capacitance network that can be placed across a 48-V d.c. voltage source. An unlimited selection of capacitors with maximum voltage ratings of 16 V each is available.



Find: The minimum number and capacitance of identical capacitors from which the network can be constructed.

Solution: Each individual capacitor can support a maximum voltage of 16 V; therefore, a series combination is required. The minimum number of capacitors needed is given by —

$$\text{number of capacitors} = \frac{\text{total voltage}}{16 \text{ V}} = \frac{48 \text{ V}}{16 \text{ V}} = 3$$

Find the capacitance of each capacitor:

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad (\text{for three capacitors})$$

$$\frac{1}{C_s} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} \quad (\text{Since } C_1 = C_2 = C_3 = C \text{ for identical capacitors})$$

$$\frac{1}{12\mu\text{F}} = \frac{3}{C}$$

$$\frac{C}{12\mu\text{F}} = 3 \quad (\text{Both sides multiplied by } C)$$

$$C = 3(12\mu\text{F})$$

$$C = 36\mu\text{F} \quad (\text{Both sides multiplied by } 12\mu\text{F})$$

Thus, three 36- μF capacitors rated at 16 V each, connected in series, are equivalent to a single 12- μF , 48-V capacitor.

The equivalent circuit of a number of capacitors connected in series has the following properties:

1. The reciprocal of the equivalent capacitance is equal to the sum of the reciprocals of the individual capacitances:

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots \text{ etc.} \quad (9)$$

2. The charge associated with each individual capacitor is the same and equal to the charge on the equivalent capacitor:

$$Q_T = Q_1 = Q_2 = Q_3 = \dots \text{ etc.} \quad (10)$$

3. The voltage across the equivalent capacitor is equal to the sum of the voltages across the individual capacitors:

$$V_T = V_1 + V_2 + V_3 + \dots \text{ etc.} \quad (11)$$

4. The potential energy E_p stored by the equivalent capacitor is equal to the sum of the energies by the individual capacitors:

$$E_p = E_1 + E_2 + E_3 + \dots \text{ etc.} \quad (12)$$

CAPACITORS IN SERIES/PARALLEL NETWORKS

A circuit that contains both series and parallel networks of capacitors often must be analyzed. This is easily done using the rules already developed, as illustrated in Example E.

EXAMPLE E: EQUIVALENT CAPACITANCE OF A SERIES-PARALLEL NETWORK	
Given:	Five capacitors are connected as in adjoining sketch.
Find:	Their equivalent capacitance.
Solution:	Start with the simplest segment, in this case the upper branch. Here there are 3 capacitors in series. Find the equivalent capacitance of this series segment:
$\frac{1}{C_s} = \frac{1}{8\mu F} + \frac{1}{6\mu F} + \frac{1}{24\mu F}$ $= \frac{3}{24\mu F} + \frac{4}{24\mu F} + \frac{1}{24\mu F} \quad (\text{common denominator is used})$	

$$\frac{1}{C_s} = \frac{8}{24\mu F} = \frac{1}{3\mu F} \quad (\text{fraction is added and reduced})$$

$$C_s = 3\mu F \quad (\text{reciprocal of both sides is taken})$$

Draw an equivalent circuit with the original segment replaced by its equivalent capacitance. The three capacitors now are in parallel. Find the equivalent capacitance of this parallel arrangement:

$$C_p = 3\mu F + 2\mu F + 4\mu F$$

$$C_p = 9\mu F$$

The equivalent capacitance of the entire network is $9\mu F$.

SUMMARY

Capacitors store and release charge and potential energy. "Equivalent capacitance" is "the capacitance of a single capacitor that can replace a group of capacitors without changing the characteristics of the circuit." Often, an equivalent circuit is more practical and economical than one containing a single large capacitor. The equivalent capacitance in parallel circuits is equal to the sum of the individual capacitances; and the total charge and total energy in the equivalent capacitor is equal to the sum of that in the individual capacitors, although the voltage across each individual capacitor is the same. In a series circuit, however, the reciprocal of the equivalent capacitance is equal to the sum of the reciprocals of the individual capacitances; and the voltage and energy in the equivalent capacitance is equal to the sum of that in the individual ones. The charge in each capacitor is the same and equal to that on the equivalent capacitor.

LABORATORY

SERIES AND PARALLEL CAPACITORS

MATERIALS

Power supply.
Perf-board with solderless spring clips.
Wire leads.
Assortment of non-electrolytic capacitors.
Vacuum tube voltmeter (VTVM).

PROCEDURE

1. Turn on the VTVM and allow it to warm up.
2. Connect three capacitors in parallel, as illustrated in Figure 4. Label Figure 4 with the capacitance values you have used.

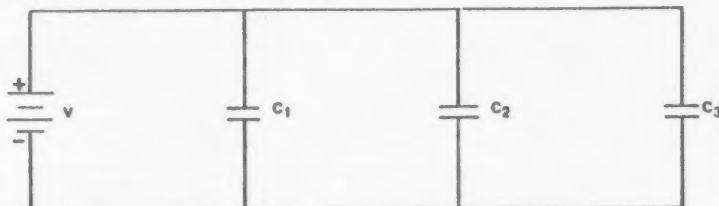


FIG. 4

3. Set the power supply voltage to about 3 V. Measure voltages across each capacitor and the voltage source. Record in Data Table 1.
4. Calculate the charge stored by each capacitor from Equation 2.
5. Calculate the energy stored by each capacitor from Equation 3.
6. Calculate the equivalent capacitance from Equation 4.
7. Calculate the total charge stored from the values of C_p and V using Equation 2.
8. Calculate the total energy stored from the values of C_p and V using Equation 3.

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9. Verify Equations 5, 6, and 7 for this circuit.
10. Reconnect the capacitors in series, as shown in Figure 5. Label Figure 5 with the appropriate values of capacitance.



FIG. 5

11. Repeat steps 3-5 for this circuit. Record results in Data Table 2.
12. Calculate the equivalent capacitance C_s from Equation 9.
13. Calculate the charge Q and the energy E of the equivalent capacitor from values of C_s and V ; use Equations 2 and 3, respectively.
14. Verify Equations 10, 11, and 12 for this circuit.
15. How does this experiment demonstrate the principles given in the Discussion section?

DETERMINATION OF UNKNOWN CAPACITANCE

MATERIALS

Same as for first experiment.

PROCEDURE

1. Connect a capacitor of unknown capacitance in series with a known capacitor across the power supply as shown in Figure 6. Record capacitance of the known capacitor in Data Table 3.

2. Set the power supply voltage to approximately 3 volts. Measure the voltage across each capacitor with the VTVM, and record these values in Data Table 3.

3. Use Equation 2 to calculate the charge Q on the known capacitor from its capacitance and the voltage across it. Since C_1 and C_2 are in series, C_2 must carry an equal amount of charge Q .

4. From the values of Q and V , calculate the unknown capacitance. Use Equation 1. Verify your results with your Instructor.

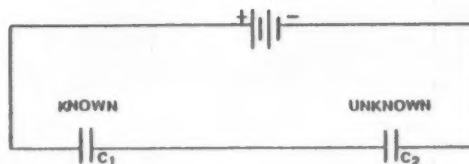


FIG. 6

DATA TABLE 1		
Source: $C_1 = \underline{\hspace{1cm}} \mu\text{F}$	$C_1 = \underline{\hspace{1cm}} \mu\text{F}$	$C_3 = \underline{\hspace{1cm}} \mu\text{F}$
$V = \underline{\hspace{1cm}} \text{V}$ $V_1 = \underline{\hspace{1cm}} \text{V}$	$V_2 = \underline{\hspace{1cm}} \text{V}$	$V_3 = \underline{\hspace{1cm}} \text{V}$
$Q_1 = C_1 V_1 = \underline{\hspace{1cm}} \text{C}$	$Q_2 = C_2 V_2 = \underline{\hspace{1cm}} \text{C}$	$Q_3 = C_3 V_3 = \underline{\hspace{1cm}} \text{C}$
$E_1 = \frac{1}{2} C_1 V_1^2 = \underline{\hspace{1cm}} \text{J}$	$E_2 = \frac{1}{2} C_2 V_2^2 = \underline{\hspace{1cm}} \text{J}$	$E_3 = \frac{1}{2} C_3 V_3^2 = \underline{\hspace{1cm}} \text{J}$
$C_p = C_1 + C_2 + C_3 = \underline{\hspace{1cm}} \text{F}$	$Q_T = C_p V = \underline{\hspace{1cm}} \text{C}$	$E_p = \frac{1}{2} C_p V^2 = \underline{\hspace{1cm}} \text{J}$

DATA TABLE 2		
Source: $C_1 = \underline{\hspace{1cm}} \mu\text{F}$	$C_2 = \underline{\hspace{1cm}} \mu\text{F}$	$C_3 = \underline{\hspace{1cm}} \mu\text{F}$
$V = \underline{\hspace{1cm}} \text{V}$ $V_1 = \underline{\hspace{1cm}} \text{V}$	$V_2 = \underline{\hspace{1cm}} \text{V}$	$V_3 = \underline{\hspace{1cm}} \text{V}$
$Q_1 = C_1 V_1 = \underline{\hspace{1cm}} \text{C}$	$Q_2 = C_2 V_2 = \underline{\hspace{1cm}} \text{C}$	$Q_3 = C_3 V_3 = \underline{\hspace{1cm}} \text{C}$
$E_1 = \frac{1}{2} C_1 V_1^2 = \underline{\hspace{1cm}} \text{J}$	$E_2 = \frac{1}{2} C_2 V_2^2 = \underline{\hspace{1cm}} \text{J}$	$E_3 = \frac{1}{2} C_3 V_3^2 = \underline{\hspace{1cm}} \text{J}$
$\frac{1}{C_5} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \rightarrow C_5 = \underline{\hspace{1cm}} \mu\text{F}$		
$Q_T = C_5 V = \underline{\hspace{1cm}}$ $E_p = \frac{1}{2} C_5 V^2 = \underline{\hspace{1cm}}$		

DATA TABLE 3	
$C_1 = \text{ \_\_\_\_\_\_ } \mu\text{F}$	
$V_1 = \text{ \_\_\_\_\_\_ } \text{V}$	$V_2 = \text{ \_\_\_\_\_\_ } \text{V}$
$Q_T = C_1 V_1 = \text{ \_\_\_\_\_\_ } \text{C}$	$C_2 = \frac{Q}{V_2} = \text{ \_\_\_\_\_\_ } \text{F} = \text{ \_\_\_\_\_\_ } \mu\text{F}$

SELF CHECK

1. Calculate the equivalent capacitance of each of the circuits in Figure 7.

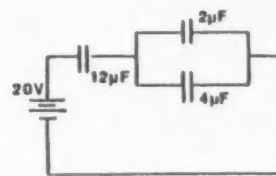
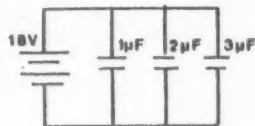
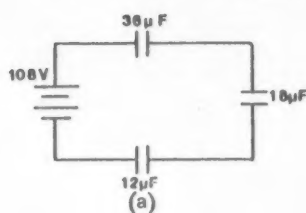


FIG. 7

2. Calculate the total charge and energy stored in each of the circuits of Figure 7.
3. Calculate the voltage across each capacitor in Figure 7a.
4. Calculate the charge and energy stored for each capacitor in Figure 7b.
5. List four properties of the equivalent circuit for each of the following:

Parallel-connected capacitors.
Series-connected capacitors.

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UNIFIED TECHNICAL CONCEPTS

MODULE 7 T 1
HEAT STORAGE SYSTEMS



THERMAL APPLICATION OF
POTENTIAL AND KINETIC ENERGY

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

Cheap and plentiful energy has been available for generations. Coal, oil and gas have always been abundantly available to heat homes, power automobiles and fuel industries. Just as the supply of these fossil fuels begins to dwindle, energy from the atom and the sun becomes important.

In the past, when all fuels were abundantly available, not much effort was made to conserve or maximize their use. As the fuel costs go up, more and more interest in making maximum use of the available energy becomes vital. Storing energy so that it can be used at a later time will greatly optimize energy usage.

This module will investigate how different materials store heat and release it at a later time when it is needed. Also investigated is the heat pump using a heated reservoir of water as a potential source.

PREREQUISITES

Student should have completed Concept Module 7-0, "Potential and Kinetic Energy."

OBJECTIVES

- Define:
 - Specific heat.
 - Heat capacity.
 - Heat pump.
 - Heat source.
 - Heat sink.
 - Coefficient of performance (COP).

- Given physical and thermal properties of a material, calculate the amount of heat that can be stored in the material for a given temperature change.
- Sketch a heat pump, labeling critical parts, and describe its operation in a short paragraph.
- Experimentally determine heat storage capabilities of a container of water, and of a container of water and rocks, given needed materials and equipment.

DISCUSSION

HEAT STORAGE CAPACITY OF MATERIALS

All materials vary in heat storage capacity or ability to store heat. One measure of this ability is the specific heat of a material, which is the number of Btu required to raise one pound of the material 1° F. For example, water has a specific heat of 1.0, which means that 1 Btu is required to raise the temperature of one pound of water 1° F. Since one gallon of water weighs 8.4 pounds, it requires 8.4 Btu to raise a gallon of water 1° F.

Different materials absorb different amounts of heat while undergoing the same temperature rise. While it takes 100 Btu to heat 100 pounds of water 1° F, it takes only 22.5 Btu to heat 100 pounds of aluminum 1° F. Specific heat of aluminum is therefore 0.225.

Specific heats of various building materials and other common materials found inside buildings are listed in Table 1. Heat capacity, or the amount of heat needed to raise one cubic foot of the material 1° F, is also listed, along with the density of each material. Although specific heat of concrete, for

TABLE 1: SPECIFIC HEATS AND HEAT CAPACITIES OF COMMON MATERIALS

Material	Specific Heat (Btu/lb x °F)	Density (lb/ft ³)	Heat Capacity (Btu/ft ³ x °F)
Water (40° F)	1.00	62.5	62.5
Steel	0.12	489	58.7
Cast iron	0.12	450	54.0
Copper	0.092	556	51.2
Aluminum	0.214	171	36.6
Basalt	0.20	180	36.0
Marble	0.21	162	34.0
Concrete	0.22	144	31.7
Asphalt	0.22	132	29.0
Ice (32° F)	0.487	57.5	28.0
Glass	0.18	154	27.7
White oak	0.57	47	26.8
Brick	0.20	123	24.6
Limestone	0.217	103	22.4
Gypsum	0.26	78	20.3
Sand	0.191	94.6	18.1
White pine	0.67	27	18.1
White fir	0.65	27	17.6
Clay	0.22	63	13.9
Asbestos wool	0.20	36	7.2
Glass wool	0.157	3.25	0.51
Air (75° F)	0.24	0.075	0.018

example, is only 1/4 that of water, its heat capacity is more than half that of water. The density of concrete compensates somewhat for its low specific heat, and concrete stores relatively large amounts of heat per unit volume.

As an illustration of these concepts, consider Example A. In reality, things are a bit more complicated. But this example gives a rough idea of how much heat could be stored in a concrete slab. (Suppose the house has 200 square feet of south windows and a solar heat gain of 1000 Btu/ft². This might be typical for a sunny, January day.) The slab can store 200 ft² x 1000 Btu/ft² or 200,000 Btu of solar energy with a temperature rise of about 10° F. The stored solar heat is then released at night to help keep the house warm while the inhabitants sleep.

EXAMPLE A: STORING HEAT IN A CONCRETE SLAB

Given: A 20 x 40 ft house with a well insulated concrete slab 9 inches thick. By 5:00 p.m. on January 21, the slab has warmed up to 75° F from sunlight flooding in the south windows. From that time until early the next morning, the outdoor temperature averages 25° F, while the indoor air averages 65° F.

Find: Temperature of the slab at 9:00 the next morning if the house is well insulated and loses heat at a rate of 300 Btu/hour/°F and there is no source of auxiliary heat.

Solution: Total heat loss from the house during that period is the product of the (rate of heat loss) times the (number of hours) times the (average temperature difference between the indoor and outdoor air).

$$\Delta H = \frac{300 \text{ Btu}}{\text{hr} \times ^\circ\text{F}} \times 16 \text{ h} \times (65^\circ \text{F} - 25^\circ \text{F})$$

$$\Delta H = 192,000 \text{ Btu.}$$

$$V = 20 \text{ ft} \times 40 \text{ ft} \times \frac{3}{4} \text{ ft} = 600 \text{ ft}^3 \text{ (volume of slab).}$$

$$Q_{\text{stored}} = \text{heat capacity} \times \text{volume.}$$

From Table 1: heat capacity of concrete = 31.7 (say 32).

$$Q_{\text{stored}} = \frac{32 \text{ Btu}}{\text{ft}^3 \times ^\circ\text{F}} \times 600 \text{ ft}^3 = 19,200 \frac{\text{Btu}}{^\circ\text{F}}$$

For a 1° F drop in its temperature, the slab releases 19,200 Btu.

Therefore, if the slab drops 10° F, from 75° F to 65° F, it will release 192,000 Btu, just enough heat to replace that lost by the house during the night. Therefore, the slab drops to a temperature of 65° F by 9:00 the next morning.

To store a large amount of heat in a given material or in a given structure, materials should be chosen that have high heat capacities - those near the top of Table I. As heat-storage devices, concrete or stone walls insulated on the outside are superior to wood framed walls having a plywood exterior and a gypsum wallboard interior with fiberglass insulation stuffed between them. For any structure or home, the larger its heat capacity the slower its response to outdoor temperature fluctuations.

HEAT PUMPS

The heat pump has served as a combination back-up and booster in a number of heating and cooling systems. It is basically a refrigeration device working in reverse. The heat pump takes heat from one location (heat source) and delivers it to another (heat sink). The heat source is cooled in the process, and the heat sink is warmed.

A heat pump can deliver about 3 times the energy required for its operation. For every 2 Btu a heat pump takes from a source, it needs the equivalent of 1 Btu of electricity for its operation. It delivers all 3 Btu to the heat sink. Thus, its coefficient of performance (COP), or the ratio of heat energy delivered to the energy required for operation, is 3. Typically, this coefficient ranges from 2.5 to 3 for good heat pumps. By contrast, electric resistance heating has a coefficient of performance equal to 1 because it delivers 1 Btu of heat for every 1 Btu of electricity expended.

When heat pumps are used in conjunction with other heating systems, the stored heat is useful over a wider temperature range. Without heat pumps, a forced warm air system would use storage temperatures from 80° F to 100° F and a hot water radiant system from 90° F to 160° F. But, with a heat pump, both systems can use 40° F storage temperatures! The heat pump takes low grade heat from storage and delivers it at higher temperature to the heat distribution system. This increased temperature range results in an increased heat-storage capacity and markedly enhanced collection efficiency. The extra Btu's that a collector can gather each year often justify the added cost of installing a heat pump.

But heat pumps require electricity for operation. About 11,000 Btu are burned at the power plant when a heat pump uses 1 kilowatt-hour of electricity (or 3400 Btu) to deliver a total of 10,400 Btu. So, including losses at the power plant and in electrical transmission, the real coefficient of performance is

closer to 1 than 3 and electricity is expensive. High electricity costs have been a major shortcoming of past heating systems that relied on heat pumps.

HEAT PUMP PRINCIPLES

As described earlier, a heat pump is a mechanical device that transfers heat from one medium to another, thereby cooling the first and warming the second. It can be used to heat or cool a body of air or a tank of water, or even the earth. The cooled medium is called the "heat source," and the warmed medium is the "heat sink." A household refrigerator is a heat pump that takes heat from the food compartments (the heat source) and dumps it in the kitchen air (the heat sink).

The heat pump transfers heat "against the grain" from cool areas to warm. This sleight-of-hand is accomplished by circulating a heat transfer fluid or "refrigerant" (such as the freon commonly used in household refrigerators) between the source and the sink and inducing this fluid to evaporate and condense repeatedly. Heat is absorbed from the source when the heat transfer fluid evaporates there. The vapor is then compressed and pumped through a heat exchanger in the sink where it condenses - releasing its latent heat. The condensed liquid returns to the heat exchanger in the source through an expansion valve that maintains the pressure difference created by the compressor.

The packaged, self-contained heat pump used in residential applications generally reverses direction of the refrigerant flow to change from heating to cooling or vice versa. A four-way valve reverses direction of flow through the compressor so that high-pressure vapor condenses inside the conditioned space when heating is needed, and low-pressure liquid evaporates inside when cooling is desired.

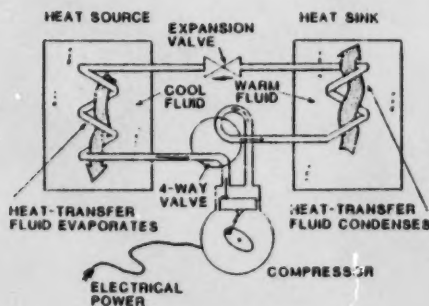


Fig. 1

Heat pumps are classified according to heat source and sink, fluid used in each, and operating cycle. The heat pump shown in Figure 1 is a water-to-water pump with reversible refrigerant flow. A household refrigerator is an air-to-air heat pump with a fixed refrigerant flow.

To illustrate these concepts, consider a heat pump that uses a basement swimming pool as a heat source for warming a

house. (The heat source illustrated in Figure 1 could be a basement swimming pool.) The following statements and drawings indicate how heat might be stored in the basement swimming pool, as much as 40,000 Btu, depending on the temperature change.

The higher the temperature a storage medium can maintain, the smaller the storage bin or tank needs to be. For instance, 1000 pounds of water can store approximately 20,000 Btu as its temperature increases from 80° F to 100° F (Figure 2).

$$\begin{aligned}\Delta H &= \text{specific heat} \times \text{weight} \times \Delta t \\ &= 1 \frac{\text{Btu}}{\text{lb} \times ^\circ\text{F}} \times 1000 \text{ lb} \times (100^\circ \text{F} - 80^\circ \text{F}) \\ \Delta H &= 20,000 \text{ Btu.}\end{aligned}$$

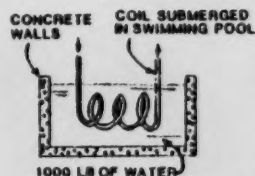


Fig. 2

The same 1000 pounds of water can store about 40,000 Btu if its temperature changes from 80° F to 120° F.

Consider a well insulated, 1200 ft² house in the Midwest (Figure 3) which loses about $\frac{10,000 \text{ Btu}}{\text{day} \times ^\circ\text{F}}$. Calculate what 15,000 pounds of water heated from 80° F to 120° F can do for this house. (Note: $10,000 \frac{\text{Btu}}{\text{day} \times ^\circ\text{F}}$ means that 10,000 Btu's are lost each day for each °F the outside temperature is cooler than the inside temperature.) House has 15,000 pounds of water in the basement. If the 15,000 pounds of water in the basement tank is heated from 80° F to 120° F, it can store approximately 600,000 Btu as shown in the following calculation.



Fig. 3

$$\Delta H = \text{specific heat} \times \text{weight} \times \text{temperature change.}$$

$$\Delta H = 1 \frac{\text{Btu}}{\text{lb} \times ^\circ\text{F}} \times 15,000 \text{ lb} \times (120^\circ \text{F} - 80^\circ \text{F}).$$

Heat capacity = 600,000 Btu.

Assuming that the heating system can use water at 80° F and that the temperature of the house is maintained at 65° F, the stored heat capacity of 600,000 Btu is equivalent to 60 degree-days for the house which loses 10,000 Btu/day x °F. This means, of course, that the heat stored in the 15,000-pound swimming pool would be dissipated in 60 days if the outside temperature were 1° F cooler than the house, namely at 64° F. On the other hand, if the average outside temperature is 5° F,

then the 600,000 Btu would maintain the house at 65° for one full day (60 degree-days ÷ 60 degrees = one day).

LABORATORY

MATERIALS

Liter beaker.
Thermometer ($^{\circ}\text{F}$).
Beaker stand.
Bunsen burner.

PROCEDURES

1. Fill beaker with about 800 milliliters of water; place on stand as in Figure 4a.
2. Raise temperature of water to about 140°F .
3. Remove heat; allow water to cool.
4. In Data Table 1, record temperature of water at regular time intervals (about every 5 minutes).
5. Continue recording temperature in data table until temperature reaches 100°F . If it takes too long for water to reach this temperature, choose another temperature, say 120°F .
6. Fill beaker with rocks, approximately 1 inch in diameter.
7. Add water until 800 milliliter level is reached (Figure 4b).
8. Repeat steps 2 through 5; record in Data Table 2.
9. Plot temperature of both of these experiments as a function of time as shown in Figure 5. Which of these systems (water or water and rocks) would you choose as a heat source if you were designing a heat pump? Explain.

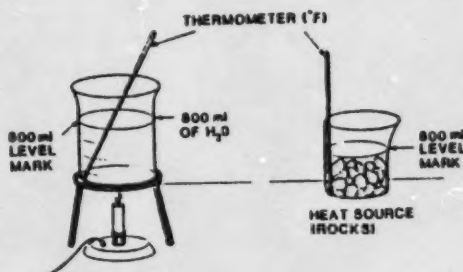


Fig. 4

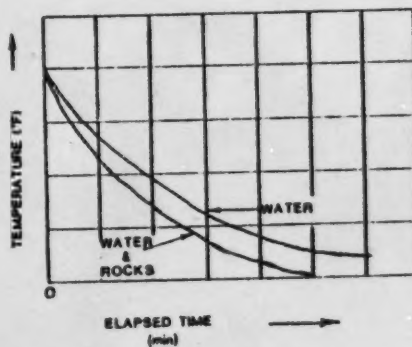


Fig. 5

SELF CHECK

1. Define:
 - Specific heat.
 - Heat capacity.
 - Heat pump.
 - Heat source.
 - Heat sink.
 - Coefficient of performance (COP).
2. How many Btu's will 10,000 lb of water store when heated from 40° F to 120° F? Specific heat of water = $\frac{1 \text{ Btu}}{\text{lb}^\circ\text{F}}$.
3. From data in Table 1, determine how many pounds of glass wool it would take to store the same Btu's as 10 pounds of water for a 10° F temperature change.
4. Sketch a heat pump, labeling critical parts of this pump.

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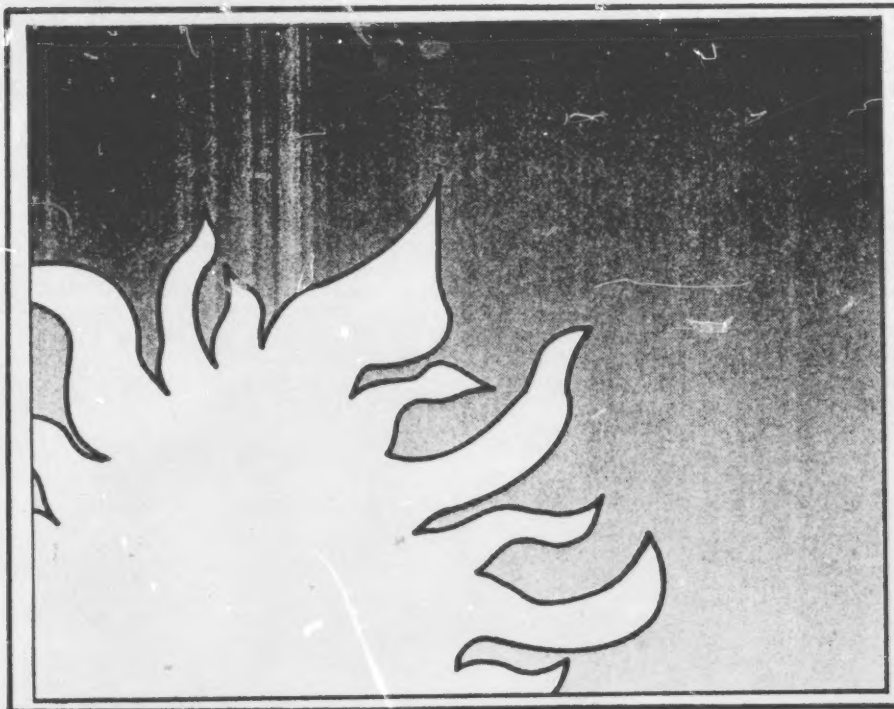
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UNIFIED TECHNICAL CONCEPTS

MODULE 7 T 2

ELECTRICAL
EQUIVALENT OF HEAT



THERMAL APPLICATION OF
POTENTIAL AND KINETIC ENERGY

Technical Education Research Center - Southwest

205

TERC - SW

INTRODUCTION

In electrical systems a portion of the electrical energy is invariably converted to kinetic energy of atoms in the form of heat. In systems designed to convert electrical energy to mechanical energy, it is desirable to reduce to a minimum the heat energy produced in this process. In other electrical systems, however, heat may be the desired result and all of the electrical energy is transformed to heat energy. In either case, it is important to understand the relationship between electrical energy and heat energy.

In this module, the process by which electrical energy is converted to heat energy will be investigated. Experiments will be used to measure the electrical equivalent of heat, that is, the energy in electrical units equivalent to a given quantity of energy in heat units.

PREREQUISITES

Student should have completed Concept Module 7-0, "Potential and Kinetic Energy."

OBJECTIVES

To complete this module, the student should be able to:

- Calculate electrical energy expended, given a quantity of charge Q and potential difference v through which that charge is moved.
- Calculate electrical energy expended, given current I , voltage V and duration of current flow t .
- Calculate heat energy gained by a material, given its mass m , initial and final temperatures T_1 and T_2 and specific heat capacity c .
- Explain in a short paragraph how electrical energy is con-

- verted to heat energy in an electrical conductor.
- Provided with necessary equipment, find energy produced, corresponding increase in heat energy of a liquid and electrical equivalent of heat of a system in which electrical energy is converted to heat energy.

DISCUSSION

Potential or voltage of a source of electrical energy is a measure of its ability to do work in moving an electric charge. One joule of energy is required to move a charge of one coulomb through a potential difference of one volt. Thus, electrical energy expended (work done) by a source of voltage in moving a given amount of charge is given as:

Electrical energy = charge x voltage.

$$\text{In symbols: } E_e = QV. \quad (1)$$

Example A illustrates the use of Equation 1 in solving a problem.

EXAMPLE A: ELECTRICAL ENERGY CALCULATION

Given: A battery with a potential of 12 volts used to move 3 coulombs of charge through a circuit.

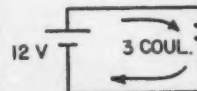
Find: How much electrical energy is used.

Solution:

$$E_e = QV$$

$$= (3 \text{ coulombs})(12 \text{ volts})$$

$$E_e = 36 \text{ C} \times \text{V} = 36 \text{ joules.}$$



All electrical energy expended by the source must appear in some other form. At least a portion of this energy is converted to heat. Figure 1 shows how electrical potential energy is converted to heat energy in an electrical conductor.

An electron at point A is accelerated by the electric field supplied by the source. The electron is accelerated toward the positive terminal, its electrical potential energy is converted

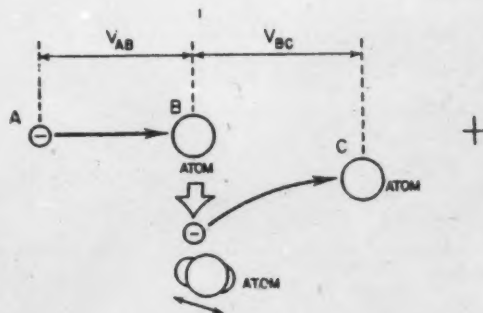


Fig. 1

to kinetic energy. In moving from point A to point B, kinetic energy of the electron is increased by the product of the electron charge and the voltage V_{AB} . At point B, the electron collides with an atom and part of its kinetic energy is transferred to the atom. The part transferred to the atom appears as vibrational energy in the material carrying the current. The electron then continues to gain kinetic energy as it accelerates toward point C, where the process is repeated. Vibrational energy imparted to the atoms of the conductor appears as heat energy, and its temperature rises. This process is called Joule heating after physicist James P. Joule, who first described it.

In a purely resistive load, all of the electrical energy is changed to heat energy. The power of an electric heating element or resistor is the rate at which electrical energy is converted to heat within the element. Since current is given by:

$$I = \frac{Q}{t}$$

and energy is $E = QV$,

$$\text{power is } P = \frac{E}{t} = \frac{QV}{t}$$

or, finally, $P = IV$,

total energy delivered by a pure resistive load is given by Equation 2.

Energy = power x time.

Energy = voltage x current x time.

In symbols: $E_e = VIt$. (2)

Example B illustrates how to use Equations 1 and 2 in solving a problem.

Recall from Concept Module 3-0, "Work," Equation 3 for the heat energy necessary to cause a temperature rise in a material.

Heat energy = mass x specific heat x change in temperature.

In symbols: $E_h = m \times c \times \Delta t$. (3)

EXAMPLE B: ENERGY AND TOTAL CHARGE CALCULATION

Given: A 220-ohm resistor connected to a 12-volt battery as shown. The switch is closed for 30 seconds.

Find: a) Energy converted to heat within the resistor and b) total charge that flows through the circuit.

Solution:

$$\begin{aligned} \text{a. From Ohm's law: } I &= \frac{V}{R} \\ &= \frac{12 \text{ volts}}{220 \text{ ohms}} \\ &= 0.0545 \text{ amps.} \end{aligned}$$

$$\begin{aligned} \text{b. From Equation 2: } E &= IVt \\ &= (0.0545 \frac{\text{C}}{\text{s}})(12 \frac{\text{joules}}{\text{C}})(30 \text{ s}) \\ &= 19.62 \text{ joules.} \end{aligned}$$

$$\begin{aligned} \text{From Equation 1: } Q &= \frac{E}{V} \\ &= \frac{19.62 \text{ joules}}{12 \text{ joules/C}} \\ &= 1.64 \text{ C} \end{aligned}$$

$$\text{or, } Q = It = (0.0545 \text{ C/s})(30 \text{ s}) = 1.64 \text{ C.}$$



1 amp = 1 C/s.
1 volt = 1 joule/C.

In practical applications of pure resistive loads in electrical circuits, actual temperature of the resistor depends on power delivered, mass and specific heat of the resistor and rate at which heat is conducted away by the environment. To make accurate measurements of the electrical equivalent of heat, it is necessary to limit investigation to systems in which heat energy can easily be distributed through a known mass of material with a known specific heat. This is most easily accomplished with liquid heating systems such as oil sump, aquarium and coffee pot heaters. If the mass of the liquid is much greater than the mass of the heating element used, heat contained in the heating element itself becomes insignificant. Example C is an application of Equation 3 to such a problem.

Since electrical energy is customarily measured in joules and heat energy is measured in calories, the relationship between electrical energy delivered in joules and heat energy gained in calories must be determined. This relationship is usually expressed as:



EXAMPLE C: CALCULATING HEAT ENERGY

Given: A small heating coil used to heat a cup of water.

Find: How much heat energy is necessary to raise the temperature of 200 g of water from 25° C to 80° C.

Solution:

$$\begin{aligned}E_h &= m \times c \times \Delta t \\&= (200 \text{ g})(1 \text{ calorie/g } ^\circ\text{C})(80^\circ \text{ C} - 25^\circ \text{ C}) \\&= (200 \text{ g})(1 \text{ calorie/g } ^\circ\text{C})(55^\circ \text{ C}) \\E_h &= 11,000 \text{ calories.}\end{aligned}$$

$$1 \text{ calorie} = 4.184 \text{ joules}$$

or

$$1 \text{ kilocalorie (thousand calories)} = 4184 \text{ joules.}$$

This relationship will be verified in the experimental portion of this module.

LABORATORY

MATERIALS

Graduated cylinder.
Centigrade thermometer.
a.c. voltmeter.
a.c. ammeter.
Stopwatch.
One-quart thermos bottle (wide mouth is best).
Water.
Electric heating element of known power rating. Note: This may be an oil sump heater, aquarium heater, the type of coil used in Example C or any similar heating element that operates on 115-V a.c. power.

PROCEDURE

1. Connect electric circuit as shown in Figure 2a and b; do not plug in circuit. Caution: Because this experiment requires high voltage, check circuit with instructor before plugging power cord into wall outlet.
2. Fill thermos bottle with cold water, about one inch from top as shown in Figure 3; insert heater and thermometer.
3. Remove heater and thermometer; pour water into graduated cylinder. Record its volume. Return water, heater and thermometer to thermos. Allow 5 minutes for water temperature to stabilize; record initial temperature of water.
4. Complete part A of data table to calculate approximate heating time necessary to cause a 10°C temperature rise in water.
5. Plug in power cord; close knife switch for appropriate time as measured with stopwatch. Record voltage and current measured. Stir water gently with thermometer. (Make sure heating element does not touch thermometer.)
6. After a 10°C rise in temperature has been attained, open switch; allow 30 seconds for temperature to stabilize; record final temperature of water.
7. Complete data table; prepare a brief analysis of probable errors in laboratory.

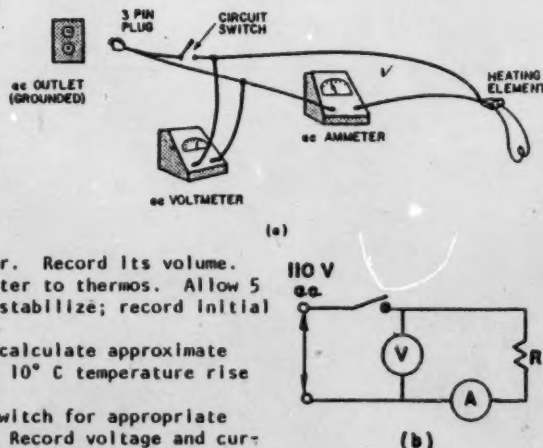
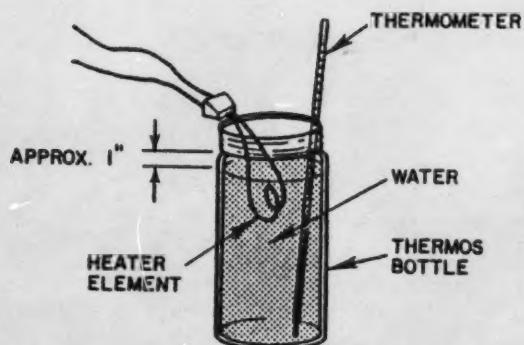


Fig. 2

Fig. 3



1 cm³ of water
has a mass of
1 g.

DATA TABLE

Part A

Volume of water: $v = \underline{\hspace{1cm}} \text{ cm}^3$.

Mass of water: $m = \underline{\hspace{1cm}} \text{ g}$.

Heat energy necessary to cause 10° C temperature rise:

$E_h = \underline{\hspace{1cm}} \text{ g} \times 1 \text{ calorie/g } ^\circ\text{C} \times 10^\circ \text{ C}$.

$= \underline{\hspace{1cm}} \text{ calories} \times 4.184 \text{ joules/calorie}$.

$E_h = \underline{\hspace{1cm}} \text{ joules}$.

Rated power of heater: $P = \underline{\hspace{1cm}} \text{ watts}$.

Time necessary to deliver electrical energy:

$t = \text{electrical energy necessary} / \text{power of heater}$.

$= \underline{\hspace{1cm}} \text{ joules} / \underline{\hspace{1cm}} \text{ joules/s}$.

$t = \underline{\hspace{1cm}} \text{ s}$.

Part B

Electrical energy.

Voltage: $V = \underline{\hspace{1cm}} \text{ volts}$.

Current: $I = \underline{\hspace{1cm}} \text{ amps}$.

Time: $t = \underline{\hspace{1cm}} \text{ s}$.

Electrical energy:

$$E_e = VIt.$$

$$= \frac{\text{joules}}{\text{C}} \times \frac{\text{C}}{\text{s}} \times \text{s}.$$

$$E_e = \text{ joules.}$$

Heat energy.

$$\text{Beginning temperature: } T_1 = \text{ }^\circ \text{C.}$$

$$\text{Final temperature: } T_2 = \text{ }^\circ \text{C.}$$

$$\begin{aligned} \text{Change in temperature: } \Delta t &= T_2 - T_1 \\ &= \text{ }^\circ \text{C} - \text{ }^\circ \text{C} \\ &= \text{ }^\circ \text{C.} \end{aligned}$$

Heat energy.

$$\begin{aligned} E_h &= m \times c \times \Delta t \\ &= \text{ g} \times (1 \frac{\text{calorie}}{\text{g} \times \text{ }^\circ \text{C}}) \times \text{ }^\circ \text{C} \\ &= \text{ calories.} \end{aligned}$$

$$\text{Electrical equivalent of heat} = E_e/E_h$$

$$= \text{ joules/ calories}$$

$$E_e/E_h = \text{ joules/calorie.}$$

$$\text{Percent error} = \frac{\text{theoretical results} - \text{experimental results}}{\text{theoretical results}}$$

$$\times 100$$

$$= \frac{4.184 \text{ J/cal} - \text{ J/cal}}{4.184 \text{ J/cal}} \times 100$$

$$= \text{ } \%$$

Error analysis notes.

SELF CHECK

1. How much electrical energy is expended when a charge of 16 coulombs is moved through a potential difference of 22.5 volts?
2. How much electrical energy is expended when a current of 0.45 amps is driven by a potential of 115 volts for one minute?
3. If energy in problem 2 is converted to heat and transferred to 1 kg of water with an initial temperature of 2°C , what is the final temperature?
4. Explain in a short paragraph how electrical energy is converted to heat energy in an electrical conductor.
5. How much time is required for a 250-watt heating element to raise the temperature of 250 g of water by 25°C ?

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UNIFIED TECHNICAL CONCEPTS

MODULE 7 T 3

MECHANICAL
EQUIVALENT OF HEAT



THERMAL APPLICATION OF
POTENTIAL AND KINETIC ENERGY

Technical Education Research Center - Southwest

TEC - SW

INTRODUCTION

Drilling holes and sawing timbers cause tools to heat up when used. A technician needs to know how to calculate the heat that will be produced in any mechanical action. Precisely how mechanical energy and heat energy are related to each other needs to be known so that steps can be taken to make machinery more efficient.

In the experimental part of this module, tiny balls of lead will be dropped through a given height and, in the process, will convert gravitational potential energy into heat energy. Ratio of the quantities (mechanical energy expended/heat energy produced) yields the "mechanical equivalent of heat."

PREREQUISITES

Student should have completed Concept Modules 7-0, "Potential and Kinetic Energy,"

OBJECTIVES

To complete this module, the student should be able to:

- Define "mechanical equivalent of heat."
- Demonstrate a knowledge of the relationship between heat and mechanical energy by stating equivalent energies in calories, joules and British thermal units.
- Calculate several problems concerning increase in temperature of a falling object.
- Calculate several problems concerning transfer of potential to kinetic energy.

DISCUSSION

The term "mechanical equivalent of heat" implies that there is a relationship between mechanical energy and heat energy. The discovery of this relationship - that heat energy is just the mechanical energy of motion of atoms and molecules - was one of the great discoveries in physics. When this became clear, the separate sciences of mechanics and calorimetry (heat exchange) were unified.

When mechanical energy is converted to heat energy (or vice versa) the principle of conservation of energy applies. Therefore, a precise relationship is found to exist between energy measured in mechanical units and energy measured in heat units. Several such conversions are given in Table 1.

TABLE 1. CONVERSION TABLE	
Heat Energy	Mechanical Energy
1 calorie	4.184 joules
1 kilocalorie	4184 joules
1 Btu (British thermal unit)	778 ft-lb or 1054 joules

Examples A, B and C illustrate the equivalence of mechanical and heat energies.

EXAMPLE A: TEMPERATURE INCREASE CALCULATION/AUTO BRAKES

Given: A 1600-kg car traveling 20 m/s is brought to a stop. Assume 40% of the heat developed goes into the brake drum. The specific heat of iron is 0.11 cal/g °C or 0.11 kcal/kg °C.

Find: Increase in temperature of the 5-kg brake drum.

Solution:

$$\text{Kinetic energy lost} = \frac{1}{2} mv^2 = \frac{1}{2} (1600 \text{ kg}) (20 \text{ m/s})^2 \\ = 320,000 \text{ joules.}$$

$$\text{Heat developed} = 320,000 \text{ J} \times \frac{1 \text{ kcal}}{4,184 \text{ J}} = 76.4 \text{ kcal.}$$

40% of this heat goes into the brake drum. The other 60% goes into heating the brake lining, tires and road.

$$0.40 \times 76.4 = 30.6 \text{ kcal.}$$

$$\text{Heat} = mc\Delta t.$$

$$30.6 \text{ kcal} = (5 \text{ kg}) (0.11 \frac{\text{kcal}}{\text{kg} \cdot ^\circ\text{C}}) \Delta t.$$

$$\text{Increase in temperature} = \Delta t = 55.6^\circ \text{C.}$$



Note: Kinetic energy could be neglected.



EXAMPLE B: TEMPERATURE INCREASE CALCULATION/WATERFALL

Given: A 50-meter waterfall. Assume that water is traveling at 2 m/s as it goes over the fall, and after it hits the bottom its speed is zero. (Assume that all mechanical energy is converted to heat energy.)

Find: Amount of temperature increase of each gram of water as it falls.

Solution: Mechanical energy = kinetic energy + potential energy.

$$ME = \frac{1}{2}mv^2 + mGh$$

$$= \frac{1}{2}(0.001 \text{ kg})(2 \text{ m/s})^2 + (0.001 \text{ kg})(9.8 \text{ m/s}^2)(50 \text{ m})$$

$$= 0.002 \text{ Nm} + 0.490 \text{ Nm}$$

$$ME = 0.492 \text{ Nm or } 0.492 \text{ J.}$$

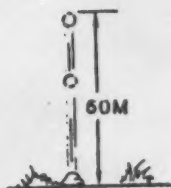
$$\text{Heat developed} = 0.492 \text{ J} \times \frac{1 \text{ cal}}{4.184 \text{ J}} = 0.118 \text{ cal.}$$

$$\text{Heat} = mc\Delta t.$$

$$0.118 \text{ cal} = (1 \text{ g})(1 \text{ cal/g } ^\circ\text{C}) \Delta t.$$

$$\text{Increase in temperature} = \Delta t = 0.118^\circ \text{C.}$$

Temperature does not increase very much because water has a high value of specific heat.



EXAMPLE C: TEMPERATURE INCREASE CALCULATION/LEADWEIGHT

Given: A 1-gram lead weight falls from rest 50 meters above the ground. Lead has a specific heat of 0.031 cal/g $^\circ\text{C}$. Assume that all mechanical energy is converted to heat energy.

Find: Its increase in temperature.

Solution:

$$\text{Potential energy} = mGh$$

$$= (0.001 \text{ kg})(9.8 \text{ m/s}^2)(50 \text{ m}) = 0.49 \text{ J.}$$

$$\text{Heat} = 0.49 \text{ J} \times \frac{1 \text{ cal}}{4.184 \text{ J}}$$

$$= 0.117 \text{ cal}$$

$$\text{Heat} = mc\Delta t.$$

$$0.117 \text{ cal} = (1 \text{ g})(0.031 \text{ cal/g } ^\circ\text{C}) \Delta t.$$

Increase in temperature = $\Delta t = 3.8^\circ \text{C}$. This temperature is much larger than that of Example B, showing that lead heats up much faster than water. It has a lower value of specific heat than water.

LABORATORY

MATERIALS

Cardboard mailing tube - about 100 cm long.
Lead shot - at least 500 grams.
Thermometer measuring to nearest tenth of a degree.
Two beakers - one larger than the other.
Small amount of ice.
Two cork stoppers for ends of cardboard mailing tube.
Triple-beam balance.
Meter stick.

PROCEDURES

In this experiment, tiny lead balls will be dropped through a given height, thereby converting mechanical energy into heat energy as in Example C. Instead of calculating increase in temperature of lead as in Example C, increase will be experimentally measured. Heat gained by the lead, in calories, will be determined by the formula:

$$\text{Heat energy} = mc (T_2 - T_1)$$

where

m = mass of lead shot in grams.
 c = specific heat of lead = 0.031 cal/g °C.
 T_1 = initial temperature in °C.
 T_2 = final temperature in °C.

Loss in potential energy, in joules, of the lead shot is calculated from the formula:

$$\text{Potential energy} = mGh$$

where

m = mass of lead shot in kg.
 G = 9.8 m/s².
 h = height through which shot falls in meters.

The mechanical equivalent of heat is calculated by the formula:

$$\text{Mechanical equivalent of heat} = \frac{\text{potential energy in joules}}{\text{heat energy in calories}}$$

The experimental value is compared with the correct value of 4.184 joules/calorie.

1. Record results in data table (use constant units).
2. Record room temperature.
3. Weigh a small beaker. Pour between 500 and 700 grams of lead shot into beaker. Weigh and subtract weight of beaker to get weight of lead shot.
4. Set beaker of lead shot in a larger beaker containing ice water. Stirring, cool shot down about 4 or 5 degrees below room temperature. Be careful to keep shot dry. Remove beaker of shot from ice water. Record initial temperature of shot.
5. Hold mailing tube in a vertical position. Insert cork in lower end. Pour lead shot into mailing tube; insert stopper into top end (Figure 1a).
6. Invert tube, allowing shot to fall to other end; repeat for a total of 98 times. Counting time shot was poured from beaker to tube (Figure 1a) and from tube to beaker (Figure 1c), this will make 100 times shot has fallen length of tube (Figure 1b).
7. Remove stopper from top end. Invert tube again, letting shot roll into beaker (Figure 1c).
8. Move thermometer gently around in shot; record highest temperature. This temperature should be a few degrees above room temperature. Check to make sure thermometer has reached its

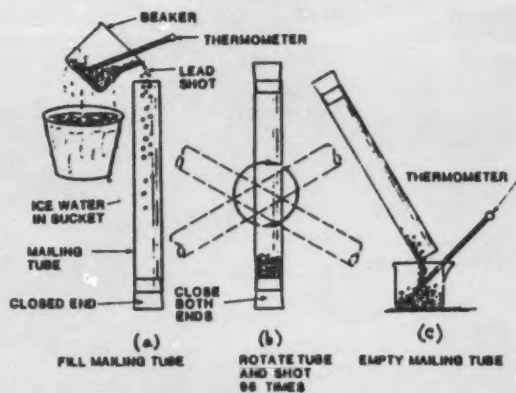


Fig. 1

peak value. This step must be done quickly, before shot begins to cool down appreciably.

9. Calculate heat (in calories) absorbed by lead shot.
10. Measure distance shot fell down tube. Multiply this number by 100 to get total height through which shot fell.
11. Calculate potential energy in joules.
12. Calculate mechanical equivalent of heat.
13. Compare with correct value of 4.184 joules/calorie.

Note: A percent error of 40% or more in this experiment is not uncommon because of the many variables that were not accounted for - such as heating air and tube. The purpose of the experiment was not to achieve a low percent of error, but to show that mechanical energy really is converted into heat. A percentage error of more than 20% is not very convincing. Repeat experiment with a slightly different initial temperature to give better results.

DATA TABLE

Room temperature ____.

Beaker weight ____.

Beaker and shot weight ____.

Lead shot weight ____.

Initial temperature of lead shot ____.

Final temperature of lead shot ____.

Calculated value of heat energy in calories ____.

Average distance lead shot fell (in one fall) ____.

Number of times tube was inverted ____.

Total height through which lead shot fell ____.

Calculated value of potential energy in joules ____.

Calculated value of mechanical equivalent of heat ____.

Correct value of mechanical equivalent of heat ____.

Percent error ____.

Note:

- a. Why is it advisable to start with the shot below room temperature and end up above room temperature?
- b. Mention some sources of error and actions that can be taken to correct errors and improve experimental results.

SELF CHECK

1. Define "mechanical equivalent of heat."
2. Demonstrate a knowledge of the relationship between heat and mechanical energy by stating equivalent energies in calories, joules and British thermal units.
3. Calculate increase in temperature of a lead object that falls 40 meters:
 - a. If object has a mass of 10 kg.
 - b. If object has a mass of 500 kg.
4. Considering answers to problem 3, calculate increase in temperature of a copper object of unknown mass that falls 6 m. Specific heat of copper is 0.093 cal/g °C.
5. A 30-gram lead bullet traveling 300 m/s is stopped by a wall. Initial temperature of bullet is 30° C. Calculate:
 - a. Kinetic energy in joules of bullet before striking wall.
 - b. Heat, in calories, that goes into bullet. Assume 90% goes into bullet and 10% goes into wall.
 - c. Final temperature of bullet. Hint: The bullet will not go above the melting temperature of lead (330° C) unless all of the lead melts.

Is there enough heat to melt some or all of the bullet? If so, how much is melted? It takes 5.9 calories to melt one gram of lead.
6. A mechanic lifts a 375-lb motor using a pulley system. A pull on the chain a distance of 10 feet with a constant force of 50 lb lifts the motor 15 inches. Calculate:
 - a. Work done by mechanic in ft x lb.
 - b. Increase in potential energy in ft x lb.

If answer to part a is greater than answer to part b, calculate amount of energy left over. Assuming the motor is at rest after it is moved, where did the energy go?

 - c. Efficiency using the following formula:

$$\% \text{ efficiency} = \frac{\text{work output}}{\text{work input}} \times 100\%$$

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See especially Chapter 11.

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UNIFIED TECHNICAL CONCEPTS

MODULE 7 T 4
THERMAL DRYING



THERMAL APPLICATION OF
POTENTIAL AND KINETIC ENERGY

Technical Education Research Center - Southwest

TERC - SW

INTRODUCTION

In this module, the concept of kinetic energy of atoms and molecules is applied to an explanation of thermal drying of moist materials. Thermal drying is an important process in many industries, including painting, building construction materials, photographic production, food preparation and agricultural industries. The thermal nature of drying is explored in a discussion of molecular behavior of water vapor in accordance with the kinetic molecular theory. Evaporation of moisture from a moisture-containing solid is discussed as an important mechanism of drying. General characteristics of several types of industrial thermal dryers are presented. The student is provided with thermal drying experiments which determine the moisture content of an apple and of green, freshly cut wood.

PREREQUISITES

Student should have completed Concept Module 7-0, "Potential and Kinetic Energy."

OBJECTIVES

To complete this module, the student should be able to:

- Define:
 - Hygroscopic material.
 - Bound moisture.
 - Unbound moisture.
 - Equilibrium moisture content.
 - Free moisture content.
- Explain the role of evaporation in drying, using the conclusions of the kinetic molecular theory of matter.
- Explain the role of moisture content of the air surrounding a substance in the drying process.
- Name and describe three industrial methods of drying; give an example of what each is used for.

- Given an initial undried mass of a substance and a later partially dried mass of the substance, calculate the moisture content of the substance.
- Given the appropriate equipment and a piece of green wood, determine its moisture content.

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DISCUSSION

Thermal drying may be defined as the removal of moisture from a material by the application of heat. Moisture is removed from the material by evaporation into the surrounding air or gas, which then carries the moisture away by convective means. The source of heat energy provides the kinetic energy by which the moisture changes from liquid to vapor. It also conditions the surrounding air or gas to receive and hold the evaporated moisture.

In the food industry, thermal drying leads to dollar savings since it reduces the weight of the dried food in the handling and moving processes. Dried food can also be transported easily without refrigeration. In the paint industry, thermal drying hastens the production process, as it also does in the paper production and coatings industry.

MOISTURE CONTENT IN MATERIAL SOLIDS

A solid may contain moisture in one of two forms, unbound or bound. Unbound moisture resides on the surface of the material and in large capillaries opening to the surface. Bound moisture is held in fine capillaries and skeletal structure of the material. A material containing both bound and unbound moisture is called a hygroscopic material. See Figure 1. For example, vegetable matter is hygroscopic.

In a material such as tanned shoe leather, the bound moisture retained may be as much as 40 percent of its dry weight. Moisture in excess of that amount would be unbound.

If a material is exposed to air at a given temperature and moisture content, it will have an equilibrium moisture content. Moisture retained in a material in excess of the equilibrium moisture content is free moisture content (the amount that could be removed if the material was indefinitely exposed to the same air state).

MOLECULAR KINETIC ENERGY IN A THERMAL DRYING PROCESS

The kinetic molecular theory states that the temperature of a substance is a measure of the average kinetic energy of its molecules and that in a substance molecules have a random distribution of

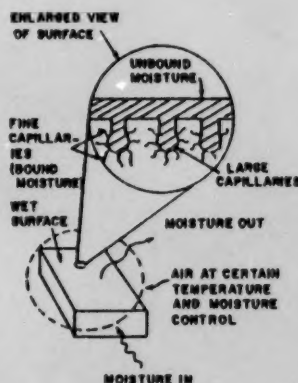


Fig. 1

velocities. Some molecules may be slow-moving; some may be fast-moving. Total energy possessed by a given substance can be regarded as the sum of the energies of all its molecules.

As kinetic energy of any one molecule depends on its mass and velocity, the total energy of a body depends upon its total mass (the number of molecules) and the average molecular velocity.

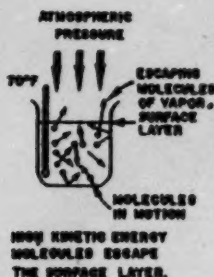


Fig. 2

A beaker of water at 70° F contains a large number of molecules (Figure 2). Some molecules at a given instant are moving slowly. Other molecules at the same instant are moving rapidly. Several of the high-energy, rapidly moving molecules escape the surface of the water.

Molecules of water vapor which escape are actually those of low-pressure steam. The latent heat of vaporization of water at the boiling point (212° F) and atmospheric pressure is 970 Btu/lb. Water is however evaporated at other temperatures requiring different amounts of heat. Table I shows various values of the latent heat of vaporization of water for temperatures less than boiling.

TABLE I. LATENT HEAT OF VAPORIZATION OF WATER AT ATMOSPHERIC PRESSURE	
Temperature (F°)	Latent Heat of Vaporization (Btu/lb)
32	1075
50	1065
60	1059
80	1048
100	1037
110	1031
:	:
212	970

In the drying process, the thermal heat source provides this energy. The rate at which evaporation occurs is governed by heat supplied to the water and also the condition of the air above the liquid (i.e., its acceptance and retention of escaped moisture). Placing a lid or cover on the beaker in Figure 2

would result in a condition shown in Figure 3. The number of vapor molecules falling back to the liquid surface equals the number escaping - the condition of equilibrium (saturation).

The rate of evaporation of a liquid can be influenced by controlling moisture content and temperature of the air above the liquid surface.

THERMAL DRYING OF A HYGROSCOPIC MATERIAL

Most hygroscopic materials require a "gentle" heat or a heat source causing the material to remain at low temperatures while drying. A drying temperature which is too high may cause a surface "crusting effect" which would slow the flow of vapor to the surface. Figure 4 illustrates a representation of a hygroscopic material being dried thermally.

To measure moisture content of a drying material, the mass of the substance must be monitored and recorded during the drying process. Equation 1 provides a simple means of determining percent moisture content.

$$\% \text{ moisture} = \frac{H_0 - H_t}{H_0} \times 100$$

where

H_0 = initial mass at beginning of drying.

H_t = mass of substance at a later time.

A problem using this equation is solved in Example A.

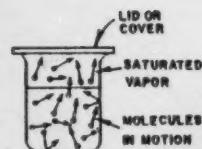
EXAMPLE A: MOISTURE CONTENT CALCULATION

Given: A prune has a mass of 5 grams prior to drying. One hour later its mass is 3 grams.

Find: The % moisture by calculation.

Solution: % moisture = $\frac{5g - 3g}{5g} \times 100 = \frac{2}{5} \times 100$

% moisture = 40%.



THE NUMBER OF VAPOR MOLECULES FALLING BACK TO THE LIQUID SURFACE EQUALS THE NUMBER ESCAPING - THE CONDITION OF EQUILIBRIUM (SATURATION)

Fig. 3

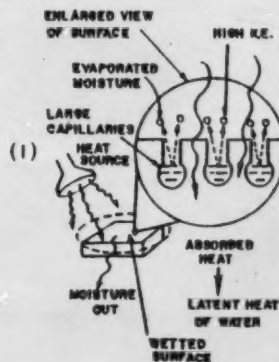


Fig. 4

At least two physical changes may occur in a substance during the drying process; the most predominant is volume shrinkage. Some substances also suffer discoloration. If shrinkage is not allowed to occur naturally, the substance may split or crack.

INDUSTRIAL METHODS OF THERMAL DRYING

Three basic methods of industrial thermal drying are conductive, radiative, and convective.

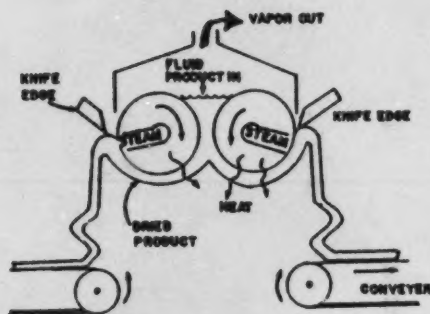


Fig. 5

In the food industry, rotating drums filled with steam are used to dry milk, fruits, purees and cereals. Drying temperatures are normally high with a drum dryer. If the material is very wet and very thin, the conductive method shown in Figure 5 may be used. Slurries can be spread over the slowly rotating, heated drums. As the drum revolves, moisture evaporates and the dried material is peeled off with a knife edge.

Electromagnetic radiation is used in paint and photographic film industries. Because many materials absorb radiant energy in the 4- to 8- μ m range of wavelength (infrared region) infrared radiant heating is widely used for drying film, coatings and sheet materials such as paper. Infrared radiation has good depth penetration in apples, as will be experimentally observed later in this module. Figure 6 shows a schematic of a film dryer.

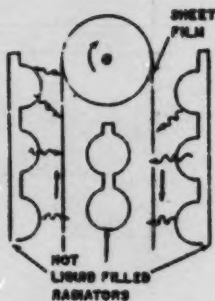


Fig. 6

The most commonly used thermal dryer is the convective type (Figure 7). Temperature and humidity of incoming warm air control the rate of moisture removal. Warm, moist air circulates over the material to be dried and is rejected, carrying the evaporated moisture from the drying cavity. Dryers of this type are used to dry peaches, apples and a variety of foodstuffs. The lumber industry uses the convective type of thermal drying to hasten the seasoning of wood.

MOISTURE CONTENT IN WOOD

Green wood (lumber just cut) contains a great deal of moisture in its cell walls. The center of the tree, known as the

heartwood, usually contains an amount of moisture different from the sapwood, that portion of the tree one to three inches below the bark. Moisture content of wood in the lumber industry is defined as the weight of water expressed as a percentage of the weight of oven-dry wood. Stated in equation form:

$$\text{Moisture content (\%)} = \frac{\left(\frac{\text{Initial weight}}{\text{oven-dry weight}} \right) - \left(\frac{\text{oven-dry weight}}{\text{oven-dry weight}} \right)}{\text{oven-dry weight}} \times 100. \quad (2)$$

Typical values of the moisture content of green wood are given in Table 2. Note that some values exceed 100%.

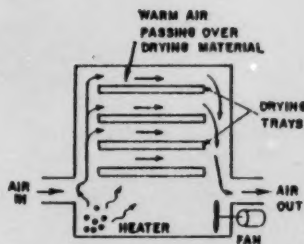


Fig. 7

TABLE 2. AVERAGE MOISTURE CONTENT OF GREEN WOODS		
Type	Heartwood (%)	Sapwood (%)
Elm, American	95	92
Hackberry	61	65
Maple, silver	58	97
Oak, red	83	75
Oak, white	64	78
Sycamore, American	114	130
Cedar, western red	58	249
Pine, western white	62	148

Example B illustrates a moisture content calculation for wood.

The purposes for drying wood (seasoning) are to reduce the weight of the wood, which reduce shipping costs, reduce or eliminate shrinkage and warping, increase strength and nail-holding ability, decrease susceptibility to fungi and attacks by insects and improve the wood's capacity to hold a preservative or paint.

Wood is normally dried to some average moisture content whose value varies depending on the part of the country in which the lumber is to be used. An average moisture content is ap-

EXAMPLE B: WOOD MOISTURE CONTENT CALCULATION

Given: A test block of wood with an initial weight of 88.7 grams is placed in a drying oven; 24 hours later it weighs 76.6 grams.

Find: Moisture content of the test block.

Solution:

$$\text{Moisture (\%)} = \frac{(88.7 \text{ grams} - 76.6 \text{ grams})}{76.6 \text{ grams}} \times 100$$

$$\text{Moisture (\%)} = 15.8\%$$

proximately 8 percent for the middle of the United States above Central Texas, 11 percent along the Gulf Coast and Southern Atlantic Coast and 6 percent in Arizona and Nevada. Moisture content at equilibrium is that moisture content at which the wood neither gains nor loses moisture. Wood virtually always is undergoing slight changes in moisture content depending on relative humidity and atmospheric temperature. The practical objective of seasoning is to reduce moisture content of green wood to one corresponding to the average atmospheric conditions to which it will be exposed.

LABORATORY

APPLE SLICE MOISTURE CONTENT

MATERIALS

Triple beam mass balance.
Laboratory stand to support balance.
Infrared (heat) lamp, reflector and clamp.
Laboratory timer (with a capacity of over 6 hours).
Moist apple.
String for support of apple.

PROCEDURE

1. Arrange experiment equipment as shown in Figure 8.
2. Before turning on lamp and starting timer, determine mass of apple slice to nearest tenth of a gram; record this value as initial mass of apple in data table.
3. Turn on lamp; start timer simultaneously. Record mass of apple slice at 20-minute intervals for several hours (6 hours if possible). A desirable termination condition is when mass of apple slice is below 20% of initial starting mass. While conducting this step (between mass readings) complete steps 4, 5, 6.
4. Record temperature of room in which experiment is conducted.
5. Calculate percent moisture for each mass determination, using equation:

$$\% \text{ moisture} = \frac{M_o - M_t}{M_o} \times 100$$

where

M_o = initial mass of apple slice.
 M_t = mass of slice at time t .

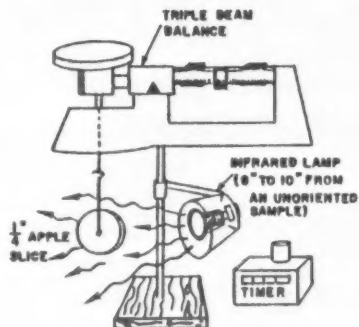


Fig. 8

- Record value in data table.
6. Prepare a graph of mass of apple slice in grams and time in minutes. Plot these points on Figure 9 after recording data in data table.
 7. Turn off lamp when finished. Leave apparatus as is for thirty or forty minutes until apple has had time to cool back to room temperature.
 8. Determine and record mass of apple slice.
 9. Leave slice in open air overnight. Determine and record mass of apple slice. What effect was evident?
 10. Describe physical characteristics of dried apple slice compared with initial slice.

DATA TABLE I

Initial mass of apple slice: _____.

Room temperature: _____.

Time (Min)	Mass (g)	% Moisture
0		
20		
40		
60		
80		
100		
120		
140		
160		
180		
200		
220		
240		
260		
280		
300		
320		
340		
360		

Mass of cool dry slice: _____.

Mass of apple slice after approximately 12 hours (overnight):

_____.

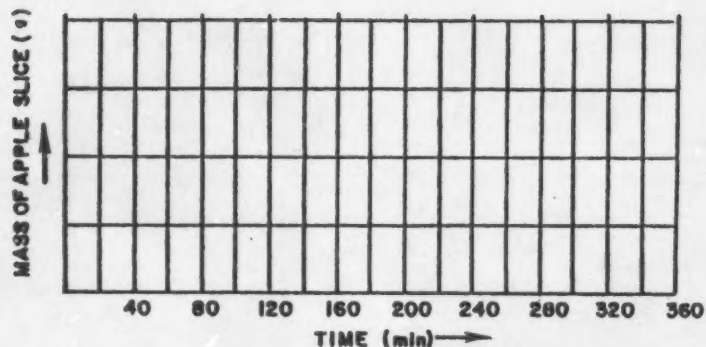


Fig. 9

MOISTURE CONTENT OF WOOD

MATERIALS

Triple beam mass balance.
 Piece of green (just cut) wood (6 to 8 inches long, 1 to 2 inches in diameter suggested size, stripped of bark).
 Electric toaster/broiler oven (or some type of laboratory oven capable of maintaining a temperature between 95° and 105°).
 Clock.
 Thermal hand mitten.

PROCEDURE

1. Arrange experiment equipment as shown in Figure 10.
2. Weigh test sample of green wood on triple beam balance; record initial mass in data table.
3. Adjust oven temperature to approximately 95° and 105°C; place wood on oven rack. Close oven door.
4. Allow a time interval of 24 hours; remove wood, using a thermal mitten to protect hand. Place wood on beam balance; record oven-dry weight in data table.

5. Calculate moisture in percent for test block using equation:

$$\text{Moisture (\%)} = \frac{(\text{Initial weight}) - (\text{oven-dry weight})}{\text{oven-dry weight}} \times 100.$$

Record this value as green wood moisture content.

6. Allow test block to remain open to air in a room of fairly constant humidity, to see if oven-dry block absorbs any moisture from air. Weigh block after another 24 or more hours to verify suspicions. Record this equilibrium moisture weight in data table; calculate equilibrium moisture content.

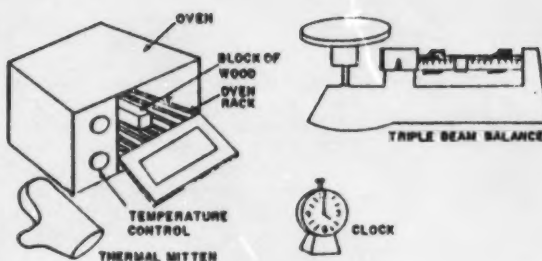


Fig. 10

DATA TABLE 2	
Description of test sample (dimensions, wood type if known):	
1.	Initial weight: _____ grams.
2.	Oven-dry weight: _____ grams.
3.	Green wood moisture content: _____ %.
4.	Equilibrium weight: _____ grams.
5.	Equilibrium moisture content: _____ %.

SELF CHECK

1. Define:
 - Hygroscopic materials.
 - Bound moisture.
 - Unbound moisture.
 - Equilibrium moisture content.
 - Free moisture content.
2. What is the relationship between temperature and the kinetic energy of molecules in a given state of matter?
3. Describe the molecular kinetic energy model which accounts for evaporation of liquids.
4. Name two conditions of the air surrounding a material being dried that affect the rate at which evaporation occurs.
5. Describe how a substance is thermally dried in a conductive drum drier.
6. Describe how a substance is thermally dried in a convective dryer.
7. Calculate moisture content of the dried apple slice after the overnight period.
8. The rate at which moisture is removed from a material is important. Using data from the module experiment and the graph produced, calculate the slope of the mass-time curve in at least three separate segments of the curve. Are the slopes (rates) the same value or different?

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UTC

UNIFIED TECHNICAL CONCEPTS

MODULE 7-T 5

MIXING LIQUIDS AT
DIFFERENT TEMPERATURES



THERMAL APPLICATION OF POTENTIAL AND KINETIC ENERGY

Technical Education Research Center - Southwest

TEC - SW

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INTRODUCTION

Thermal (heat) energy is the kinetic energy possessed by the moving molecules of a substance. The more heat that is applied to a liquid, the faster its molecules move and the higher its temperature becomes (except when a change of state to or from a solid, a liquid, or a gas is involved). When a heated liquid contacts a colder one, heat moves to the colder one because the molecules of the hotter one collide with the slower-moving molecules of the colder one until all molecules reach an average speed somewhere between their original speeds. Liquids have this kind of unseen kinetic (motion) energy, which also can be considered as potential (stored) energy in the form of heat because it can be made to perform work.

In many industries, the temperature of mixed liquids is critical, as for example in chemical process plants and petrochemical refineries. Engineers and technicians in such plants must calculate or determine the final temperature when two or more substances of different masses and temperatures are mixed, or how much heat must be applied to change the temperature of a mixture by a given amount.

This module discusses the internal kinetic energy of liquids and several methods of determining specific heat and final temperature by the process of mixing substances at different temperatures. In the laboratory, the student will determine the specific heat of a liquid and the final temperature of a mixture.

PREREQUISITES

Student should have completed Concept Module 7-0, "Potential and Kinetic Energy."

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define thermal energy, specific heat, equilibrium, and the difference between temperature and heat.
- Describe the law of the conservation of energy.
- Given the masses and temperature of two liquid samples, the specific heat of one sample, and their final temperature when mixed, calculate the specific heat of the second sample.
- Given the masses, temperatures, and specific heats of two or more samples of liquids, calculate the final temperature reached when the samples are mixed.
- Determine mathematically the final temperature of a mixture when the masses and specific heats of both samples are given. Given the appropriate equipment, verify the above calculation for the same samples and temperatures.

THERMAL ENERGY

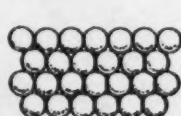
Thermal energy, sometimes called "internal energy," is the energy possessed by a substance because of the motion of its molecules. Motion is a feature of molecules in all matter. Even in the atom, the smallest particle to which an element can be reduced and still retain its original characteristics, motion occurs in the form of electrons orbiting around the nucleus. When atoms of one element combine with those of another to form molecules and clusters of molecules, the result is molecules and clusters that move incessantly in random directions. The range of their movement depends upon the state of the material in which they are found; that is, upon whether they are in a solid, a liquid, or a gas.

Figure 1 shows the arrangement of molecules in (a) a solid, (b) a liquid, and (c) a gas. Molecules in a solid move or vibrate only within a very limited region, as if held in place by tiny, powerful springs. In a solid, the molecules are so close, and the cohesion (attraction) is so strong

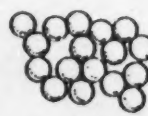
between them, that they resist being moved from their limited regions of motion. In the liquid state, the molecules, moving with greater average energies, are able to overcome the cohesion and orderly arrangement found in a solid. So the molecules are free to move in random directions until they collide with other molecules, deflect, and continue motion in a different direction. In a gas, the molecules are more energetic and much farther apart than in the solid or liquid state, which explains why a gas is easily compressed.

On the other hand, a solid or a liquid is only slightly compressible because its molecules are already so close together that they resist being forced even closer. The molecules of a liquid can slip and slide over one another with relative ease, yet they cannot occupy the place of other molecules. This is why a liquid will assume the shape of any container into which it is placed, yet will not be compressed to any degree.

The rapid motions and collisions of the molecules in a liquid cannot be seen directly because of the extremely tiny size of the molecules. However, if microscopic particles of gold, pollen, or other materials are placed within a liquid, the particles can be observed with a microscope to dart about swiftly in all directions because the molecules of the liquid are striking the particles at great speed. The added particles, although tiny themselves, are



a. MOLECULES IN A SOLID



b. MOLECULES IN A LIQUID



c. MOLECULES IN A GAS

FIG. 1



FIG. 2

thousands or millions of times greater in size and weight than the molecules which collide with them and accelerate them to such speeds. This kind of particle movement within liquids is known as Brownian motion (named after Robert Brown, a Scottish botanist, who discovered it in 1827). Figure 2 illustrates the random directions of particles in Brownian motion. In the same manner as the molecules of a liquid collide with particles and transfer energy to them, they also collide with themselves. When the liquid is heated, the particles increase their speeds in proportion to the added heat energy. This means that as the speed of the molecules increases, the temperature increases. There is a relation, therefore, between the speed of the molecules, the temperature, and the heat energy of the liquid.

In a liquid whose temperature has reached a state of equilibrium (has stabilized), the continuing collisions of molecules does not cause a further change in the temperature. When a hot liquid is mixed with a cold one, however, the faster-moving molecules of the hot liquid collide with the slower-moving molecules of the cold one, and transfer energy until an average speed (temperature equilibrium) is reached.

TEMPERATURE AND HEAT

Temperature and heat are not the same physical concepts. The difference between the two can be demonstrated easily as follows: If one beaker contains one ounce of water at 100°C and another beaker contains one gallon of water at the same temperature, the heat contained by the gallon of water is much greater than that of the ounce of water, because if the contents of each beaker is poured upon a different block of ice, the gallon of hot water will melt much more ice than the ounce of hot water. Thus, while the temperatures of the two quantities of water are the same, their heat energies clearly are not.

All molecules within a liquid at a stabilized temperature do not move at the same speed. For instance, one molecule may be traveling at great speed until struck by another molecule. The collision may cause both to lower their speeds until again struck by other molecules and speeded up. While some molecules are slowed, others are speeded up; however, the speed of millions of molecules in a liquid averages out in a predictable way. The temperature of a substance is a measure of the average kinetic energy of its molecules. Heat energy, on the other hand, is the total kinetic energy of all molecules added together, which explains why one gallon of water contains more heat energy than one ounce, although the molecules in both have the same average speeds; that is, the same temperature.

Heat energy can be absorbed or transferred to other bodies. In Figure 3, heat energy applied by the burner to the beaker of water is absorbed by the water, causing a rise in water temperature. At the same time, the heat energy of the water is increasing and escaping through the beaker and into the surroundings. The amount of heat that water must absorb from the burner to raise the water temperature by one degree ($^{\circ}\text{C}$) depends upon the mass, or weight, of the water. Twice as much time is required to raise the temperature of two ounces of water by a given amount as would be required for one ounce of water because twice as much heat energy must be transferred.

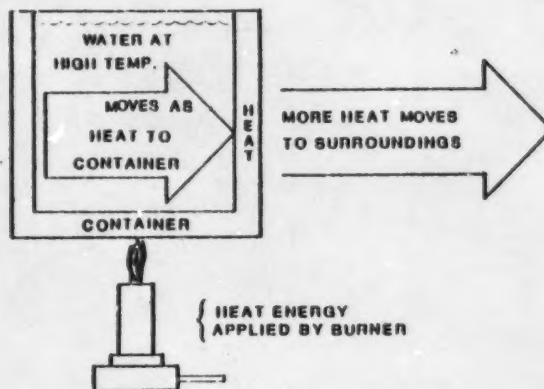


FIG. 3

SPECIFIC HEAT

More heat energy is required to heat some substances than others. This fact is an important one in industry, where heat processes are used and where energy conservation is required. Technicians often must predict in advance how much heat is required to raise the temperature of substances by any given amount. This amount of heat can be calculated when the specific heat of the substances to be heated is known. The specific heat of a substance is the amount of heat energy in calories required to raise the temperature of one gram of the substance one degree Celsius. The specific heat of most substances is known and is available in reference works. Table 1 lists the specific heats of several common substances. When the specific heat of a substance (most often a mixture) is not known, the value must be determined with a laboratory calorimeter (Figure 4). The calorimeter consists of a thin-walled metal cup situated inside of, but insulated from, an outer can. Air space of insulation usually separates the cup from the can. Necessary accessories include a thermometer, a rubber stopper, and a stirrer.

TABLE 1. SPECIFIC HEATS OF COMMON SUBSTANCES	
Substance	Specific Heat ($\frac{\text{cal}}{\text{gm} \cdot ^\circ\text{C}}$)
Water	1.0
Aluminum	0.22
Copper	0.09
Ethyl Alcohol 50% (100 proof whiskey)	0.80
Isopropyl Alcohol	0.66
Acetone	0.56
Brine 25%	0.80
Gasoline	0.50 - 0.56
Chloroform	0.28
Ethylene Glycol (auto antifreeze)	0.61

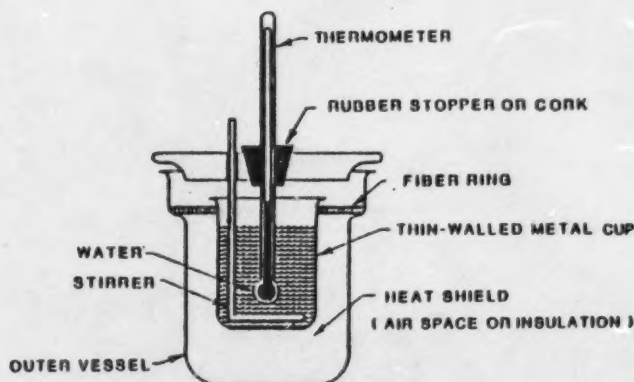


FIG. 4

as it cools down again to the original temperature. Water is widely used in industry as a heat-transfer medium; for example, it is heated and pumped through pipes to radiators many feet away, where it gives up its heat to warm homes and businesses. Water also is used in solar energy systems as a means of storing and recovering heat from the sun.

The specific heats of some common substances listed in Table 1 are made use of in practical applications. When the specific heat

The specific heat of water is $1.0 \text{ cal/gm} \cdot ^\circ\text{C}$, which means that one calorie of heat energy is required to raise the temperature of one gram of water by one Celsius degree. Water has a higher specific heat than most other substances, which indicates that more heat must be applied to it to achieve a desired increase in its temperature. However, once the temperature of water has been elevated, more heat is stored in the water and is available for transfer. In fact, the same amount of heat can be extracted from the water (or from any other substance)

of a substance is known, technicians can predict in advance how much heat will be required to raise the temperature of the substance by a given amount, as long as the mass of the substance is known. Equation 1 is used to calculate the amount of heat required to bring about a given temperature change.

$$H = mc\Delta T \quad (1)$$

where

H = quantity of heat required for a given temperature change.

m = mass of substance.

c = specific heat of substance.

ΔT = desired change in temperature.

THE LAW OF THE CONSERVATION OF ENERGY

The law of the conservation of energy states, "Energy can be changed in form but cannot be created or destroyed." Heat is a form of energy; therefore, no heat energy is lost when substances of unequal temperatures are mixed. The heat given off by the hot substance equals the heat received by the cold one, assuming that no heat is lost to the surroundings because of poor insulation.

Figure 5 illustrates how half a cup of cold coffee is warmed when an equal amount of hot coffee is added to it. The heat energy of the hot coffee is transferred, not only to the cold coffee, but also to the cup and to the surroundings. The total heat transferred is the same amount as given up by the hot coffee.

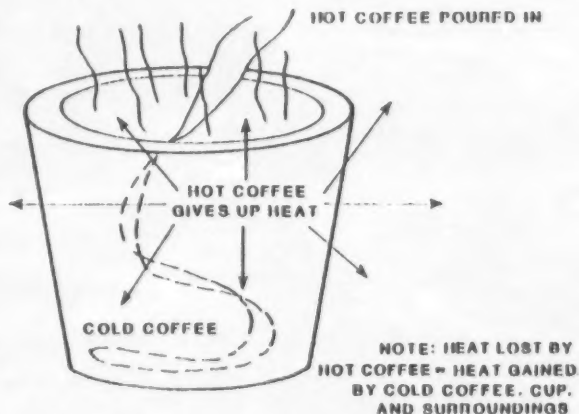


FIG. 5

Thus, when two substances of different temperatures are mixed, the heat given off by the hot one equals the heat received by the cold one. This principle is stated in Equation 2 as follows:

$$m_1 c_1 (T_1 - T_f) = m_2 c_2 (T_f - T_2) \quad (2)$$

where

- m_1 = mass of substance 1.
- c_1 = specific heat of substance 1.
- T_1 = initial temperature of substance 1.
- m_2 = mass of substance 2.
- c_2 = specific heat of substance 2.
- T_2 = initial temperature of substance 2.
- T_f = final temperature of mixture.

Equation 2 is rearranged in Example A in order to find the specific heat of a substance. It is rearranged in Example B to find the equilibrium, or final, temperature of a mixture.

EXAMPLE A: CALCULATION OF THE SPECIFIC HEAT OF A SUBSTANCE BY THE METHOD OF MIXTURES.

Given: A substance (water) with a mass of 100 g, a specific heat of 1.0 at a temperature of 20°C. A substance with unknown specific heat, a mass of 200 g and at a temperature of 80°C is mixed with the water. After the two substances have been mixed, the final temperature is 40°C.

Find: Specific heat of the second substance. (No heat loss is assumed.)

Solution: Rearrange Equation 2 to find c_2 (specific heat of second substance):

$$\begin{aligned} c_2 &= \frac{m_1 c_1 (T_1 - T_f)}{m_2 (T_f - T_2)} \\ &= \frac{(100 \text{ g})(1.00 \text{ cal/g } ^\circ\text{C})(20^\circ\text{C} - 40^\circ\text{C})}{200 \text{ g} \times (40^\circ\text{C} - 80^\circ\text{C})} \\ c_2 &= 0.25 \text{ cal/g } ^\circ\text{C} \end{aligned}$$

EXAMPLE B: CALCULATION OF THE FINAL TEMPERATURE OF A SUBSTANCE BY THE METHOD OF MIXTURES.

Given: A substance (water) with a mass of 100 g, a specific heat of 1.0, at a temperature of 20°C. A second substance with a mass of 200 g, a specific heat of 0.25, and a temperature of 80°C is mixed with the water.

Find: Final temperature of the mixture. (No heat loss is assumed.)

Solution: Rearrange Equation 2 to find T_f (final temperature):

$$T_f = \frac{m_1 c_1 T_1 + m_2 c_2 T_2}{m_1 c_1 + m_2 c_2}$$

or

$$T_f = \frac{(100)(1)(20^\circ\text{C}) + (200)(0.25)(80^\circ\text{C})}{(100)(1) + (200)(0.25)}$$

$$T_f = 40^\circ\text{C}$$

SUMMARY

Substances possess internal kinetic energy in the form of the microscopic motion of their molecules. When liquids at different temperatures are mixed, molecular motion causes them to reach an equilibrium temperature. This final temperature and the heat-energy transfer are important in industrial applications and often must be determined before the liquids are mixed.

LABORATORY

DETERMINATION OF THE SPECIFIC HEAT OF A LIQUID SAMPLE

MATERIALS

Water calorimeter constructed from two styrofoam cups (large) as shown in Figure 6.

Thermometer.

Tripple beam balance.

Bunsen burner.

2 beakers.

Water.

Isopropyl alcohol.

PROCEDURE

1. Construct calorimeter from two large styrofoam cups. Leave one cup intact. Cut 1/4 from top of other cup, leaving bottom 3/4 for use as top of calorimeter, as in Figure 6.
2. Place approximately 1/2 styrofoam cup of water in one of the beakers, and heat to near boiling. Turn off burner.
3. Weigh and record in Data Table I the mass of styrofoam cup and lid.
4. Pour the hot water into cup, place lid on cup, and weigh again to determine mass of water. Record.
5. Measure water temperature. Record.
6. Place approximately 1/2 styrofoam cup of isopropyl alcohol in the other beaker, weigh, and measure temperature. Record.
7. Pour alcohol into the cup of water, place lid on cup, place thermometer through lid hole into solution, and stir contents with thermometer.

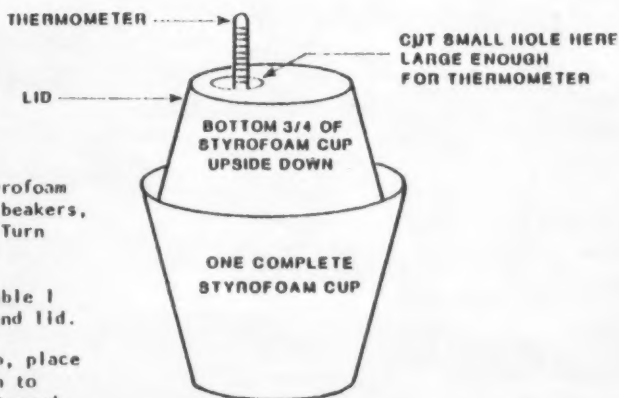


FIG. 6

8. Weigh beaker in which alcohol was contained, and subtract its mass from that of alcohol and beaker combined. Record mass of alcohol.
9. Allow about a minute for the mixture to reach equilibrium. Measure final temperature. Record.
10. Calculate specific heat of Isopropyl alcohol from Equation 2 in Example A, which reads -

$$c_2 = \frac{m_1 c_1 (T_1 - T_f)}{m_2 (T_f - T_2)}$$

NOTE: To simplify this procedure for a practical lab setup, no heat loss to container or surroundings is assumed. Actually, little heat is lost; however, when an exact determination is required, a laboratory calorimeter as in Figure 4 must be used. Heat lost to the inner calorimeter cup is taken into account by including a term representing heat absorbed by the calorimeter cup, as in the following equation:

$$c_2 = \frac{(m_1 m_1 + m_c c_c) (T_1 - T_f)}{m_2 (T_f - T_2)}$$

where

m_c = mass of inner calorimeter cup.
 c_c = specific heat of inner cup.

11. Compare results obtained in Step 10 with specific heat of Isopropyl alcohol given in Table 1. Account for difference.

DETERMINATION OF FINAL TEMPERATURE OF MIXTURE

MATERIALS

Same materials as in first experiment.

PROCEDURE

1. Follow Steps 1-4 of first laboratory procedure. However, use different masses and temperatures, and record in Data Table 2.
2. Allow about a minute for the mixture to reach equilibrium. Measure final temperature, and record in Data Table 2, "final"

temp." column, under "meas."

3. Check results obtained in Step 2 by calculating final temperature using the specific heat of isopropyl alcohol as determined in first lab experiment. Use Equation 2 as rearranged in Example B for final temperature. This equation, again, reads -

$$T_f = \frac{m_1 C_1 T_1 + m_2 C_2 T_2}{m_1 C_1 + m_2 C_2}$$

4. Enter calculated final temperature in Data Table 2 under "calculate."
5. Note the difference, if any, between measured and calculated values of the final temperature. If the measured value is slightly lower, this indicates that some heat was lost to the surroundings (cup, lid, air, etc.).

DATA TABLE 1

Test liquid	mass cup & lid	mass cup, lid & water	mass water (m_1)	temp. water (T_1)	temp. alcohol (T_2)	mass beaker & alcoh.	mass beaker	mass alcohol (m_2)	final temp. (T_f)	specific heat (c_1 & c_2)
1 water										(given) $c_1 = 1.0$
2 isopropyl alcohol										(calculate) (c_2)

DATA TABLE 2

Test liquid	mass cup & lid	mass cup, lid & water	mass water (m_1)	temp. water (T_1)	temp. alcohol (T_2)	mass beaker & alcoh.	mass beaker	mass alcohol (m_2)	final temp. (T_f)	specific heat (c_1 & c_2)
1 water									(calculate)	(given) $c_1 = 1.0$
2 isopropyl alcohol									(meas.)	(as in Data Table 1) $c_2 = 0.66$

SELF CHECK

1. Define the following terms:

Thermal energy.
The difference between temperature and heat.
Specific heat.
Equilibrium.

2. Explain the law of the conservation of energy as applied to mixtures.
3. One substance has a mass of 100 g, a specific heat of 0.80 cal/gm - $^{\circ}\text{C}$, and a temperature of 20°C . This substance is mixed with a second substance that has a mass of 200 g and temperature of 70°C . Final temperature of the mixture is 37°C . Calculate the specific heat of the second substance.
4. One substance has a mass of 95 g, a specific heat of 1.0, and a temperature of 25°C . A second substance with a mass of 210 g, a specific heat of 0.80, and a temperature of 75°C is mixed with the first. Calculate the final temperature of the mixture.

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